Geometric Misfitting in Structures – An Interval-Based Approach

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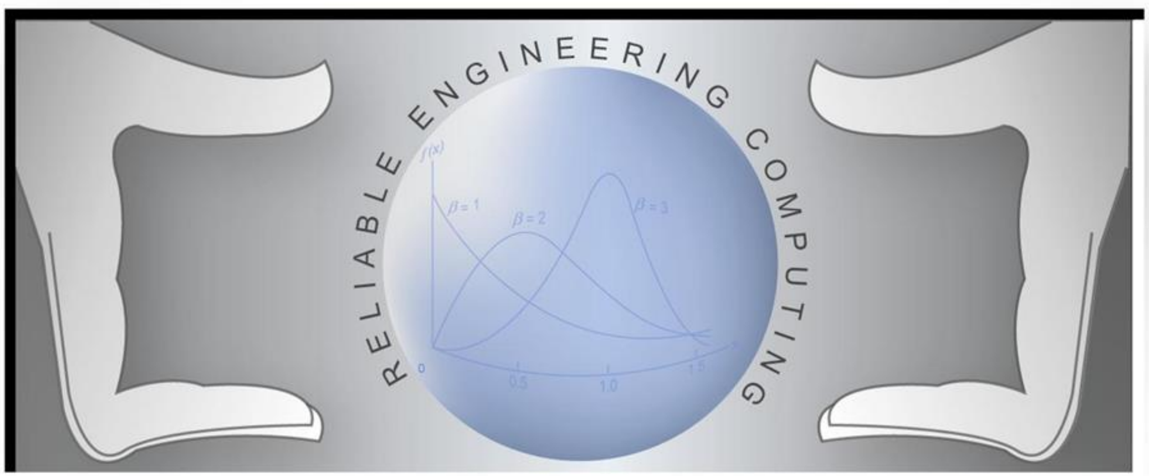
RELIABLE ENGINEERING COMPUTING
f(x)
β = 1
β = 2
β = 3
0
0.5
1.0
1.5

Outline

- **Misfit of truss members**
  - Geometric Uncertainty
  - Earlier work
- Present work
  - Uncertainty of geometric, load and stiffness properties
- Interval FEM
  - Sharpness of derived quantities
  - Mixed IFEM formulation
  - Iterative Enclosure
- Example Problems
- Conclusions

Misfit of Structural Members

- Due to fabrication errors and/or thermal changes certain bars can have improper length.
- In practice, the bar is forced into its position between two joints by applying some initial extension or compression.
- Under such a condition, some axial forces are introduced in the bars in the absence of external loads.

Geometric uncertainty

- Uncertainty in the bar length as the range between the lower and upper bounds on the nominal length of the bar.

$$L \in \boldsymbol{L}, \quad \boldsymbol{L} = [L_o - \delta L, L_o + \delta L]$$
$$\boldsymbol{L} \equiv [\underline{L}, \overline{L}] := \{L \in R \mid \underline{L} \le L \le \overline{L}\}$$

- Geometric uncertainty is defined by specifying the percentage variation of the misfit of a member about the value of the nominal length of member.

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Earlier Work..

- Muhanna, Erdolen and Mullen (2006) addressed the problem of geometric uncertainty by considering a linear interval system of equations with interval right hand side in the form $[K]\{\boldsymbol{U}\} = \{\boldsymbol{P}\}$
- Misfit forces were Solution vector of interval displacements was obtained as $\{\boldsymbol{U}\} = [K]^{-1}\{\boldsymbol{P}\}$
- However, sharp bounds to interval axial forces could not be obtained then owing to the problem of overestimation.

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Present work

- The present work obtains sharp bounds to interval displacements and axial forces by adapting the mixed finite element formulation developed by the authors.
- Truss structures with uncertainty in misfit, external loading and stiffness solved to illustrate the approach.

Interval Finite Element Method–
Error in secondary quantities

Conventional finite element method gives

$$\Pi = \frac{1}{2}\{U\}^T[K]\{U\} - \{U\}^T\{P\}$$

subject to the conditions $$\frac{\partial \Pi}{\partial \{U\}} = \{0\}$$

Secondary quantities such as force/ stress/ strain calculated from interval displacements show significant overestimation of interval bounds.

Mixed interval finite element formulation

- The potential energy functional is rewritten as $[C]\{U\} = \{0\}$
- Unknowns associated with coincident nodes are forces to have identical values. Thus we have $\Pi^* = \frac{1}{2}U^T KU - U^T P + \lambda^T (CU - V)$
- Equating the first variation of Π^* to zero,

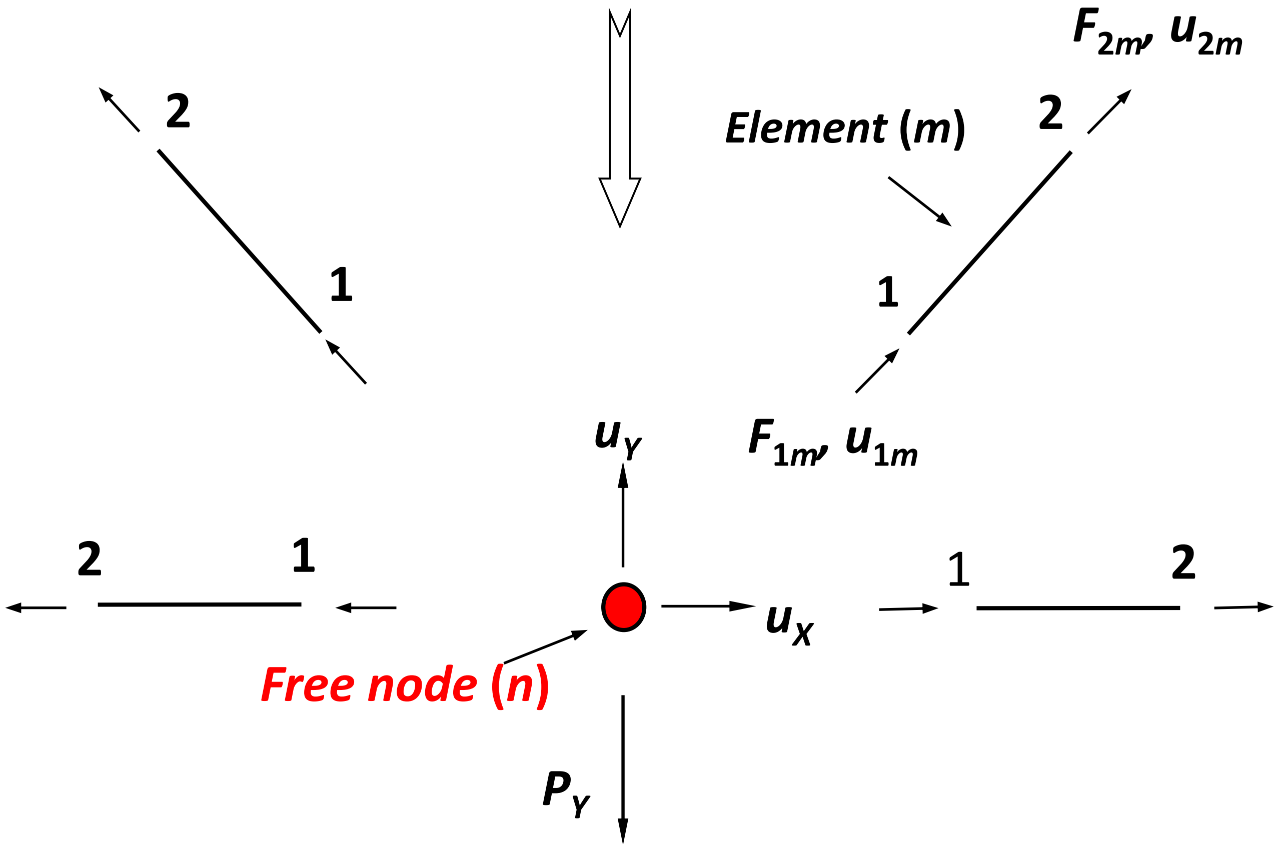
Mixed interval finite element formulation

$$\Pi^* = \frac{1}{2}U^T K U - U^T P + \lambda^T (CU - V)$$

where λ is the vector of Lagrange multipliers, $\boldsymbol{P}$ is the vector of applied interval loads and $\boldsymbol{\delta}$ contains interval multiples of additional loads due to misfit

$$\begin{pmatrix} \boldsymbol{K} & \mathrm{C}^{\mathrm{T}} \\ \mathrm{C} & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{U} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \boldsymbol{P} \\ 0 \end{pmatrix} + \begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\delta} \\ 0 \end{pmatrix}$$

Mixed interval finite element formulation



Elimination of overestimation

- Overestimation is eliminated by
  - Keeping individual elements separate and connected to free nodes and applying constraints on displacements of element nodes coincident with each free node
  - Applying misfit forces along element local axes and thus avoiding their transformation from local axes to global axes
  - Obtaining sharp enclosure to solution using Neumaier's algorithm
  - Secondary variables are part of solution vector and thus are obtained at the same level of sharpness as the primary variables

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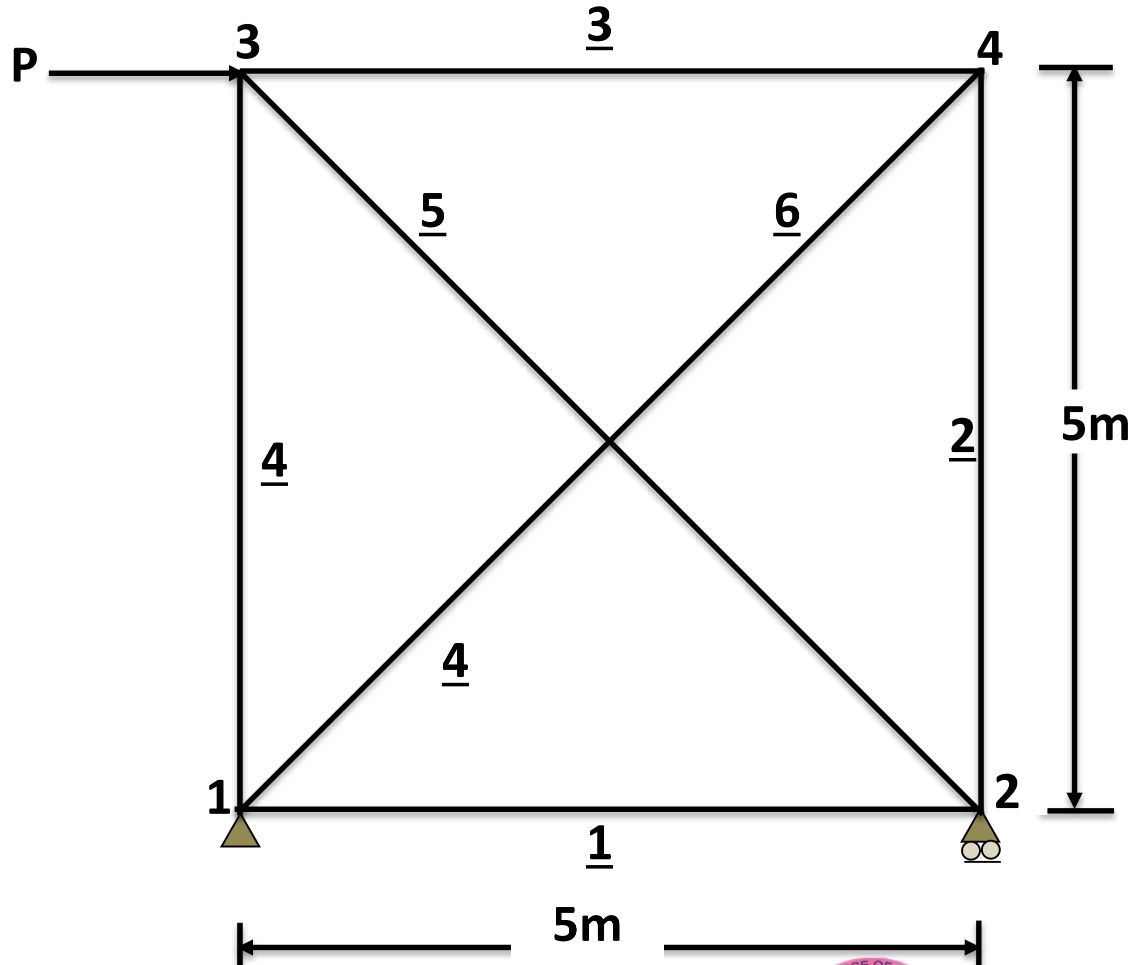
Example Problem-1 Six bar truss



Six bar truss - displacements for 0.2 percent geometric uncertainty alone						
	$U_3 \times 10^{-3}$(m)		$V_3 \times 10^{-3}$(m)		$U_4 \times 10^{-3}$(m)	
	Lower	Upper	Lower	Upper	Lower	Upper
Combin-atorial approach	-3.41421	3.41421	-1.49999	1.49999	-3.20710	3.20710
Present approach	-3.41421	3.41421	-1.50000	1.50000	-3.20710	3.20710
Error %(bounds)	0.00	0.00	0.00	0.00	0.00	0.00

Six bar truss – axial forces for 0.2 percent geometric uncertainty alone						
	N_2 (kN)		N_3 (kN)		N_6 (kN)	
	Lower	Upper	Lower	Upper	Lower	Upper
Combinatorial approach	-200.0	200.0	-141.42	141.42	-141.42	141.42
Present approach	-200.0	200.0	-141.42	141.42	-141.42	141.42
Error %(bounds)	0.0	0.0	0.0	0.0	0.0	0.0

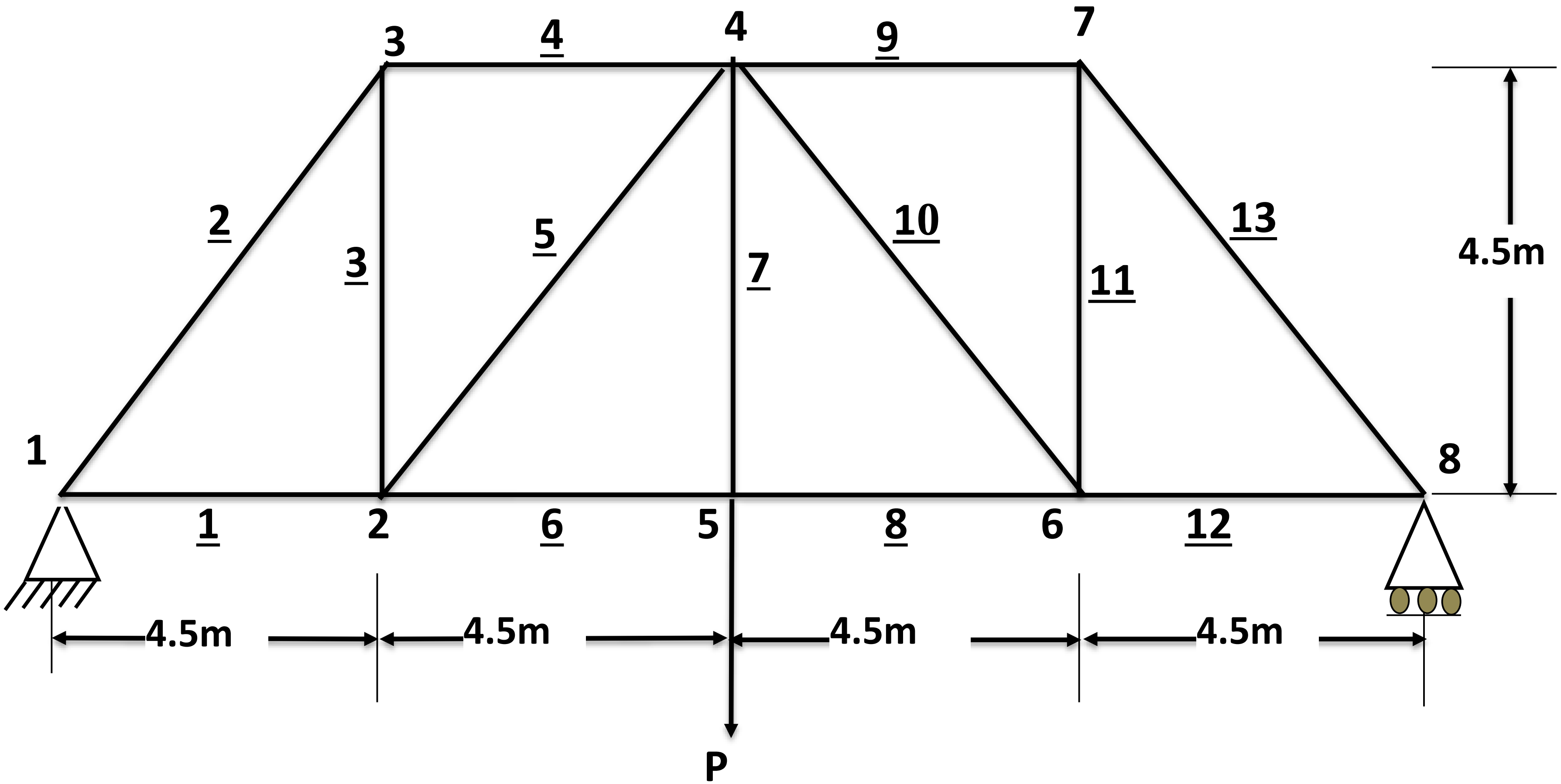
Six bar truss - displacements for 0.2 percent geometric uncertainty and 10 percent load uncertainty						
	$U_3 \times 10^{-3}$(m)		$V_3 \times 10^{-5}$(m)		$U_4 \times 10^{-5}$(m)	
	Lower	Upper	Lower	Upper	Lower	Upper
Combin-atorial approach	-2.26746	4.68167	-1.26250	1.76249	-2.29785	4.21206
Present approach	-2.26746	4.68167	-1.26250	1.76250	-2.29785	4.21206
Error %(bounds)	0.00	0.00	0.00	0.0005	0.00	0.00

Six bar truss – axial forces for 0.2 percent geometric uncertainty and 10 percent load uncertainty						
	N_2 (kN)		N_3 (kN)		N_6 (kN)	
	Lower	Upper	Lower	Upper	Lower	Upper
Combinatorial approach	-274.24	132.82	-193.92	93.92	-93.92	193.92
Present approach	-274.24	132.82	-193.92	93.92	-93.92	193.92
Error %(bounds)	0.0	0.0	0.0	0.0	0.0	0.0

Six bar truss - displacements for 0.2 percent geometric uncertainty,10 percent load uncertainty and 1 percent uncertainty in modulus of Elasticity (E)						
	$U_3 \times 10^{-3}$(m)		$V_3 \times 10^{-5}$(m)		$U_4 \times 10^{-5}$(m)	
	Lower	Upper	Lower	Upper	Lower	Upper
Combin-atorial approach	-2.28453	4.69874	-1.26999	1.76999	-2.31389	4.22810
Present approach	-2.35837	4.77259	-1.29639	1.79639	-2.38431	4.29853
Error %(bounds)	3.23	1.57	2.07	1.49	3.04	1.66

Six bar truss – forces for 0.2 percent geometric uncertainty,10 percent load uncertainty and 1 percent uncertainty in modulus of Elasticity (E)						
	N2 (kN)		N3 (kN)		N6 (kN)	
	Lower	Upper	Lower	Upper	Lower	Upper
Combinatorial approach	-275.91	134.45	-194.96	94.94	-94.94	194.96
Present approach	-279.57	138.15	-197.89	97.89	-97.68	197.68
Error %(bounds)	1.32	2.74	1.50	3.10	2.88	1.39

Example problem 2- Thirteen bar Truss



Thirteen bar truss - displacements for 0.2 percent geometric uncertainty alone						
	$U_3 \times 10^{-2}$(m)		$V_5 \times 10^{-3}$ (m)		$V_6 \times 10^{-2}$ (m)	
	Lower	Upper	Lower	Upper	Lower	Upper
Combin-atorial approach	-4.91421	4.91421	-8.82842	8.82842	-5.62132	5.62132
Present approach	-4.91421	4.91421	-8.82842	8.82842	-6.12132	6.12132
Error %(bounds)	0.0	0.0	0.0	0.0	0.0	0.0

Thirteen bar truss - displacements for 0. 2 percent geometric uncertainty and 10 percent load uncertainty						
	$U_3 \times 10\text{-}2$		$V_5 \times 10\text{-}3$		$V_6 \times 10\text{-}2$	
	Lower	Upper	Lower	Upper	Lower	Upper
Combin-atorial approach	0.79107	1.90892	-5.89350	-3.65061	-2.93409	-1.48869
Present approach	0.79107	1.90892	-5.89350	-3.65061	-2.93409	-1.48869
Error %(bounds)	0.0	0.0	0.0	0.0	0.0	0.0

Thirteen bar truss – axial forces for 0.2 percent geometric uncertainty and10 percent load uncertainty						
	N_2 (kN)		N_5 (kN)		N_6 (kN)	
	Lower	Upper	Lower	Upper	Lower	Upper
Combinatorial approach	-148.49	-134.35	-148.49	-134.35	190.00	210.00
Present approach	-148.49	-134.35	-148.49	-134.35	190.00	210.00
Error %(bounds)	0.0	0.0	0.0	0.0	0.0	0.0

Thirteen bar truss - displacements for 0.2 percent geometric uncertainty,10 percent load uncertainty and 1 percent uncertainty in modulus of Elasticity (E)						
	$U_3 \times 10^{-2}$(m)		$V_5 \times 10^{-3}$(m)		$V_6 \times 10^{-2}$(m)	
	Lower	Upper	Lower	Upper	Lower	Upper
Combinatorial approach	0.79140	1.90859	-5.89285	-3.65125	-2.93377	-1.48901
Present approach	0.78004	1.91995	-5.91647	-3.62763	-2.94518	-1.47760
Error %(bounds)	1.43	0.594	0.400	0.646	0.389	0.766

Thirteen bar truss – axial forces for 2 percent geometric uncertainty,10 percent load uncertainty and 1 percent uncertainty in modulus of Elasticity (E)						
	N2 (kN)		N3 (kN)		N6 (kN)	
	Lower	Upper	Lower	Upper	Lower	Upper
Comb-inatorial approach	-148.492	-134.350	95.000	105.000	190.000	210.000
Present approach	-148.918	-133.923	94.698	105.301	189.263	210.737
Error %(bounds)	0.287	0.317	0.317	0.287	0.387	0.350

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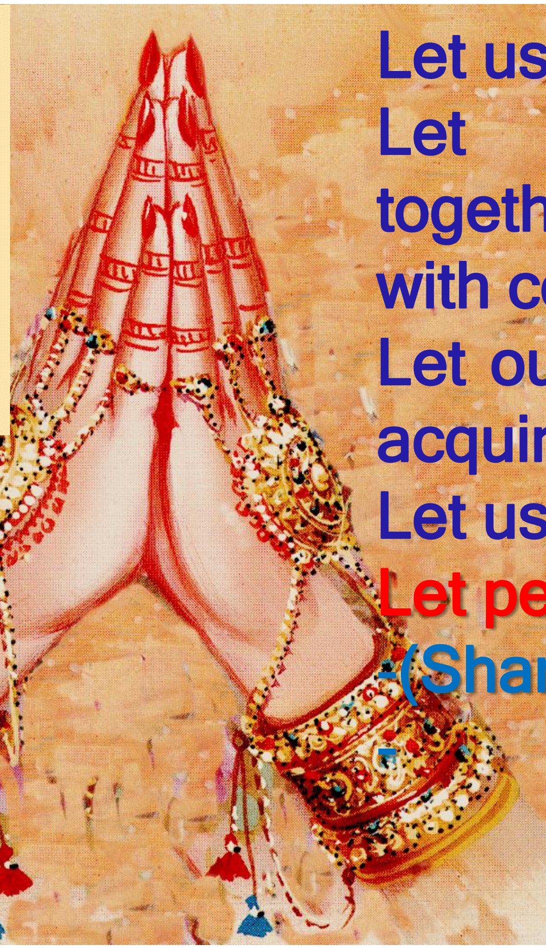
  - Iterative Enclosure
- Results
- **Summary**

Summary

- A method based on the use of a mixed finite element formulation for the analysis of structures with geometric, load and stiffness uncertainties is presented.
- Axial forces are obtained at the same level of sharpness as displacements in the simultaneous presence of geometric, stiffness and load uncertainties.
- Exact enclosure on the deformed geometry is obtained

सह नाववतु ।
सह नौ भुनक्तु ।
सह वीर्यं करवावहै ।
तेजस्वि नावधीतमस्तु ।
मा विद्विषावहै ॥
ॐ शान्तिः शान्तिः शान्तिः ॥



Let us all be together.
Let us share everything together. Let us carry out tasks with collective effort.
Let our minds work together to acquire knowledge.
Let us not harm each other.
Let peace pervade Universe.
-(Shanti Mantra ~3000 BC
- INDIA)

THANK YOU