TECHNISCHE UNIVERSITÄT DRESDEN

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IIT Chicago

Institute for Structural Analysis

Structural design with polymorphic uncertainty models

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DRESDEN concept

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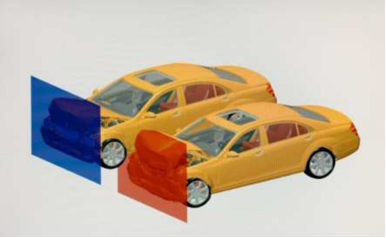
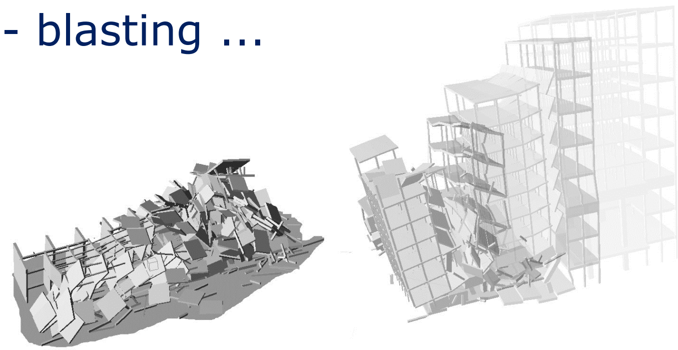
Content

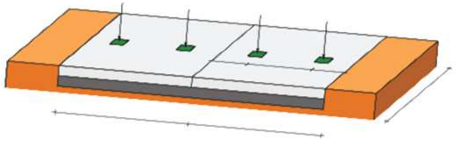


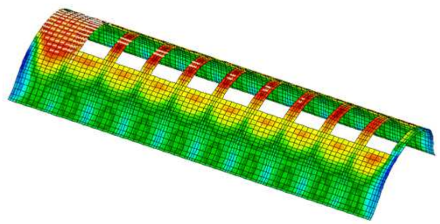
Introduction

structural analysis and design

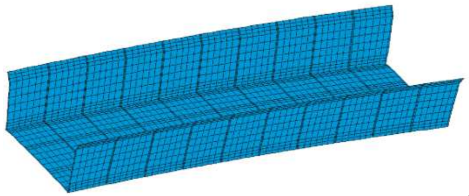
- short-term behaviour
  - wind, earthquake
  - impact
  - blasting ...







- long-term behaviour
  - aging and damage
  - creep and fatigue
  - weathering, chemical attack, ...



Introduction

problems concerning data uncertainty

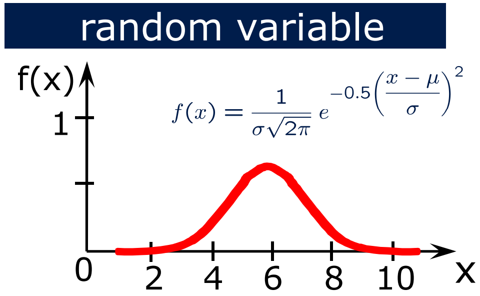
- geometry, material and loading data available to a limit extent only
- uncertain observations of geometry, material, and loading data
- varying reproduction conditions during sampling
- information deficit

consequence: polymorphic uncertainty models

Data uncertainty

name	mapping	database	characteristics
random variable	$X : \Omega \rightarrow \mathbb{R}$ $P : \Sigma \rightarrow [0; 1]$	deterministic, variability	…y

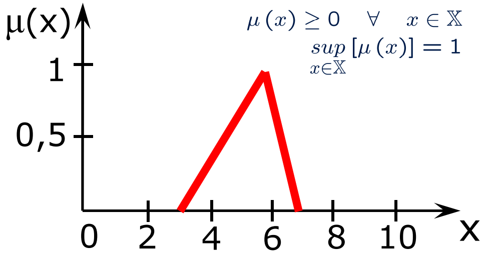
1	**6.16**
2	**5.23**
3	**6.95**
4	**6.47**
⋮	⋮
N	**5.85**



Data uncertainty

name	mapping	database	characteristics
random variable	$X : \Omega \rightarrow \mathbb{R}$ $P : \Sigma \rightarrow [0; 1]$	deterministic, variability	aleatoric, a great many
fuzzy variable	$\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0; 1]$	subjective,	epistemic, a few

fuzzy variable



$\mu(x) \geq 0 \quad \forall \quad x \in \mathbb{X}$

$\sup_{x \in \mathbb{X}} [\mu(x)] = 1$

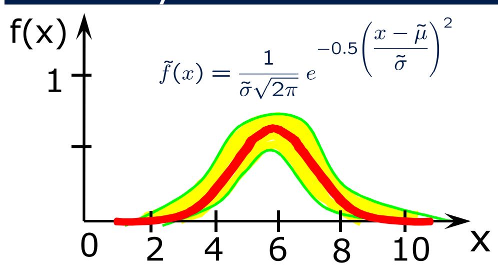
Data uncertainty

name	mapping	database	characteristics
random variable	$X : \Omega \rightarrow \mathbb{R}$ $P : \Sigma \rightarrow [0;1]$	deterministic, variability	aleatoric, a great many
fuzzy variable	$\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0;1]$	subjective,	
fuzzy probability based randomness	$X : \Omega \rightarrow \mathbb{R}$ $\hat{P} = (\hat{P}_{\beta})_{\beta \in (0;1]}$	deterministic, a few	ss

1	5.73
2	6.19
3	6.73
4	4.67
5	6.03

representation of generalized uncertainty model variables

fuzzy-random variable



$\mathcal{D}(Q, R)$ set of uncertain quantities

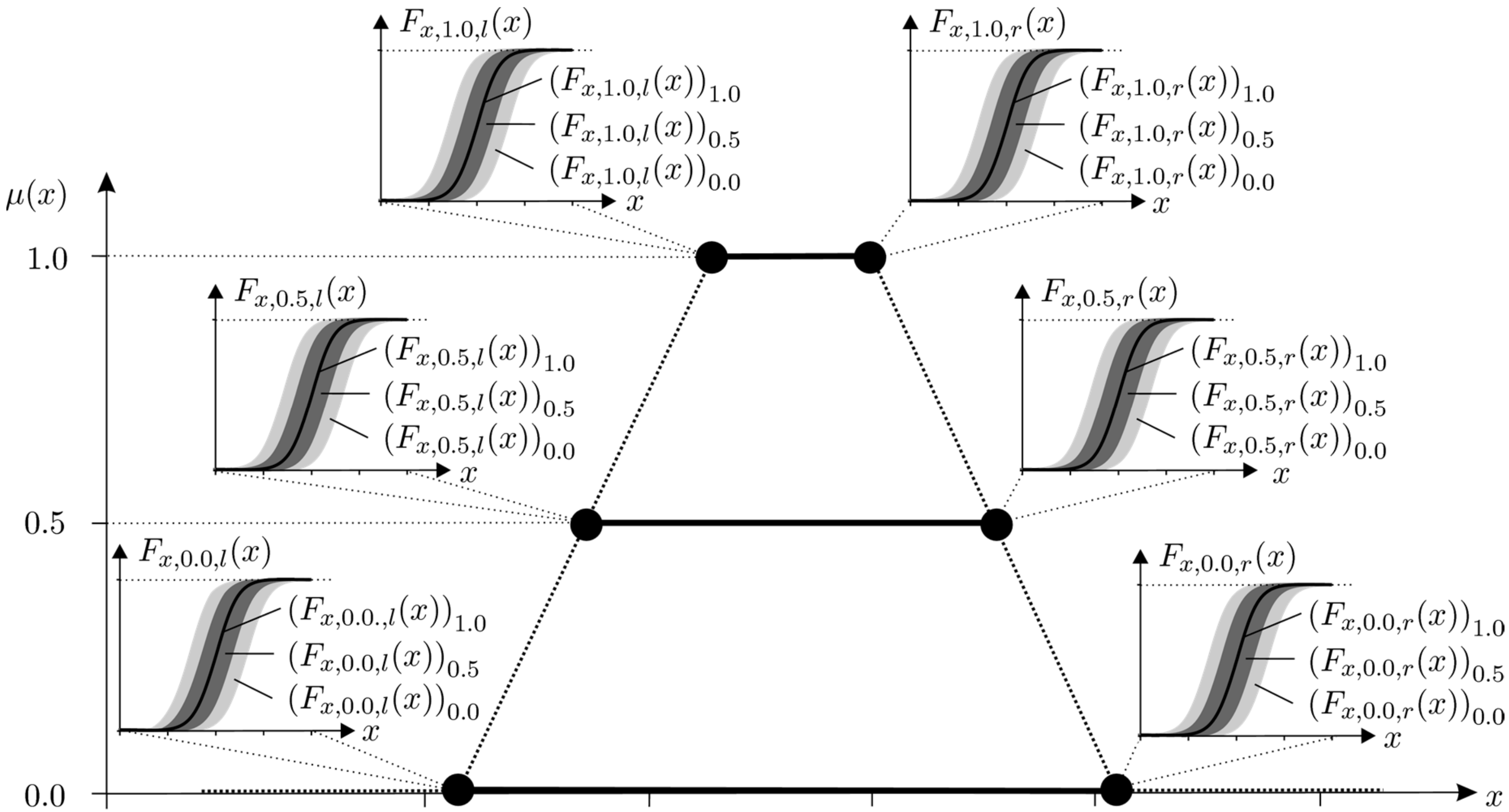
e.g. $\mathcal{D}(\Omega, \mathbb{R})$ set of random numbers

$\mathcal{D}(\mathbb{R}, [0;1])$ set of fuzzy numbers

Polymorphic uncertainty

name	mapping	data	characteristic
random variable	$X : \Omega \rightarrow \mathbb{R}$ $P : \Sigma \rightarrow [0;1]$	deterministic, very many	aleatoric (variability)
fuzzy value	$\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0;1]$	subjective, few	epistemic
fuzzy random variable	$X : \Omega \rightarrow \mathcal{F}(\mathbb{R})$ $P : \Sigma \rightarrow [0;1]$	imprecise, (very) many	variability, impreciseness
fuzzy probability based random variable	$X : \Omega \rightarrow \mathbb{R}$ $\hat{P} = \left(\hat{P}_\beta\right)_{\beta \in (0;1]}$	deterministic, some	variability, incompleteness
fuzzy probability based fuzzy random variable	$X : \Omega \rightarrow \mathcal{F}(\mathbb{R})$ $\hat{P} = \left(\hat{P}_\beta\right)_{\beta \in (0;1]}$	imprecise, some	variability, impreciseness, incompleteness

Fuzzy probability based fuzzy random variable



Numerical design with uncertain data

structural analysis $\tilde{\underline{x}}(\underline{t}) \rightarrow \tilde{\underline{z}}(\underline{t})$

e.g. $\underline{\tilde{M}} \cdot \Delta\underline{\ddot{\tilde{v}}}(\tau) + \underline{\tilde{D}} \cdot \Delta\underline{\dot{\tilde{v}}}(\tau) + \underline{\tilde{K}}_T \cdot \Delta\underline{\tilde{v}}(\tau) = \Delta\underline{\tilde{F}}(\tau)$

structural design

design parameters, a priori parameters
objective(s), constrains

design methods

- investigation of variants
- optimization (e.g. wait and see, here and now)
- solution of inverse problem (e.g. cluster methods)
- evaluation

Objective function with uncertain quantities

- deterministic objective function → deterministic optimization task

$$f_{\mathcal{Z}} : X \subset \mathbb{R}^n \to \mathbb{R} \qquad\qquad z_{min} = \min_{x \in E} f_{\mathcal{Z}}(x)$$

- objective function with uncertain quantities

$$\hat{f}^u_{\mathcal{Z}} : \mathcal{D}(Q,R) \to \mathcal{D}(V,W)$$

uncertain input quantities

uncertain design variables

uncertain a priori parameters

transformation $\mathcal{T}$

deterministic
design variables d

uncertain a priori parameters p^u

$$\Longrightarrow f^u_{\mathcal{Z}} : \mathbb{R}^{n_x} \times \mathcal{D}(Q_p, R_p) \to \mathcal{D}(V,W) : (d,p^u) \mapsto \hat{f}^u_{\mathcal{Z}}(\mathcal{T}(d),p^u)$$

General description

- **task**

  determine $L \subseteq X_d^+$ such that $\forall\, d \in X_d^+$ and $d_{min} \in L$ it holds:

  $f_{\mathcal{Z}}^u(d_{min},p^u)$ is **„lower or equal than“** $f_{\mathcal{Z}}^u(d,p^u)$ ➡ $\mathbf{minimum}_{d \in X_d^+} f_{\mathcal{Z}}^u(d,p^u)$

- **permissible range**

$$X_d^+ = \{d\ in \mathbb{R}^n \mid \forall\, j \in \{1;\ldots;a_g\} : g_j^u(d,p^u)\ \text{„lower than“}\ 0$$
$$\wedge\ \forall\, k \in \{1;\ldots;a_h\} : h_k^u(d,p^u)\ \text{„equal“}\ 0\}$$

  $g_j^u : \mathcal{D}(Q,R) \rightarrow \mathcal{D}(V,W)$ $h_k^u : \mathcal{D}(Q,R) \rightarrow \mathcal{D}(V,W)$

- **surrogate problem**

  - **passive approach** (wait-and-see)

    determine for all **p (realization of p^u)** that set $L \subseteq X_d^+$, such that $\forall d\ \in X_d^+$ and $d_{min} \in L$ it holds:

    $\mathbf{f}_{\mathcal{Z}}(\mathbf{d}_{\mathbf{min}},\mathbf{p}) \leq \mathbf{f}_{\mathcal{Z}}(\mathbf{d},\mathbf{p})$

  - **active approach** (here-and-now)

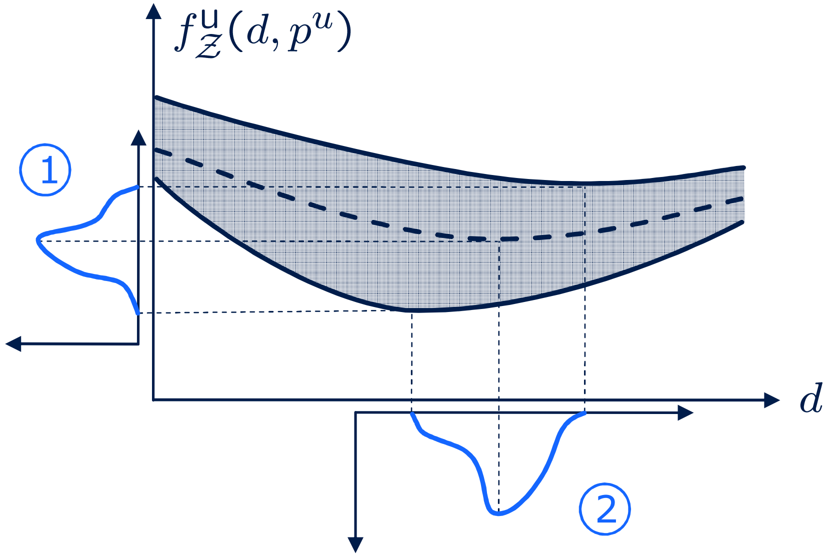
    determine $L \subseteq X_d^+$ such that $\forall\, d \in X_d^+$ and $d_{min} \in L$ it holds:

    $\mathcal{M}\left(\mathbf{f}_{\mathcal{Z}}^{\mathbf{u}}(\mathbf{d}_{\mathbf{min}},\mathbf{p}^{\mathbf{u}})\right) \leq \mathcal{M}\left(\mathbf{f}_{\mathcal{Z}}^{\mathbf{u}}(\mathbf{d},\mathbf{p}^{\mathbf{u}})\right)$

    $(\mathcal{M} : \mathcal{D}(V,W) \rightarrow \mathbb{R})$

Passive and active approach

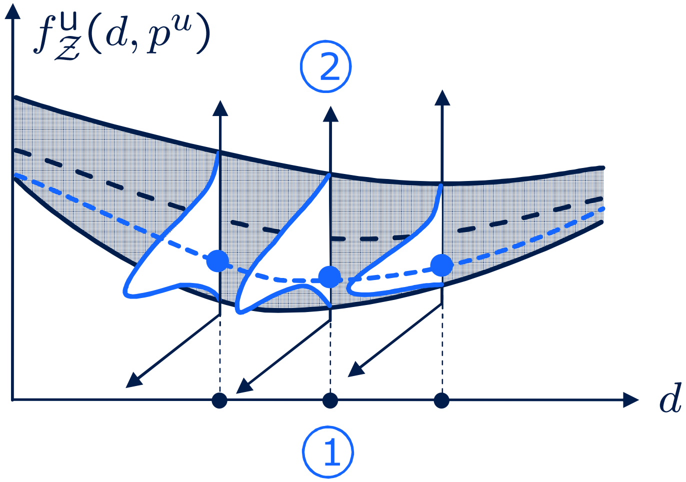
- **passive approach**



1. minimum of all deterministic objective functions
2. set of design variables

➡ **range of minimal results**

- **active approach**



1. deterministic design variables
2. uncertain result values

➡ **different design objectives**
(e.g. reliability, durability, robustness)

Summary of passive and active approach

- **passive approach**
  (wait-and-see)
  - uncertain minimum, uncertain design parameters
  - no uncertain response parameters
  - checking of constraints in post computation
  - fast detection of optimal ranges

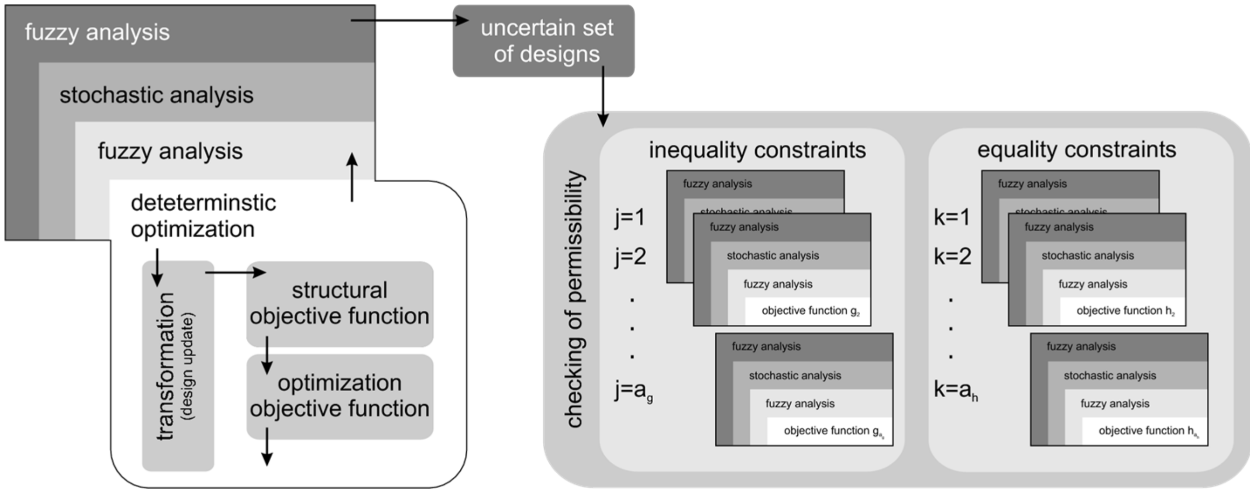
- **active approach**
  (here-and-now)
  - deterministic minimum, deterministic design parameters
  - early reduction of information
  - checking of constraints directly
  - expensive detection of the optimum

sequential application

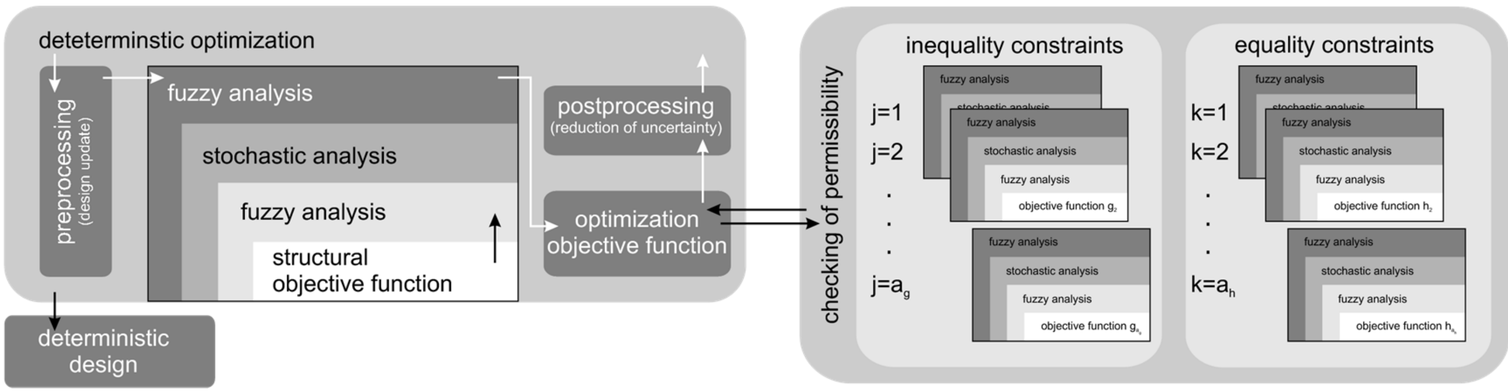
early design stage, application of fuzzy model ⊕ **final design stage, application of random based models**

Numerical frameworks

passive approach



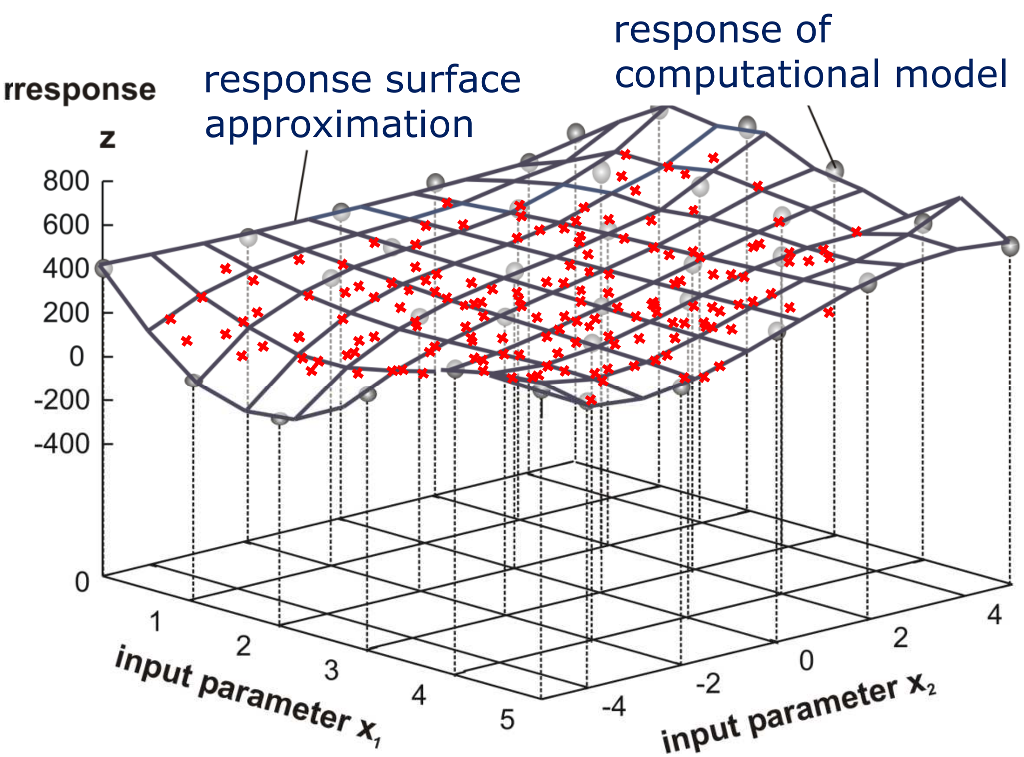
active approach



Surrogate models and reduction methods

model reduction	condensation techniques compression techniques substructure techniques
sensitivity analysis	derivate/variance based global/local
parallelization	
surrogate models	response surface methods (meta-models) „intelligent" approaches
„model free" approach	computational intelligence artificial neural networks

Principal functionality of meta-models



RSA … response surface approximation

4-step procedure to build a metamodel

design of experiments (DOE)

e.g. grid-like, random

surrogate model

e.g. polynomial, ANN

model fitting

e.g. regression, training

model validation

Example 1

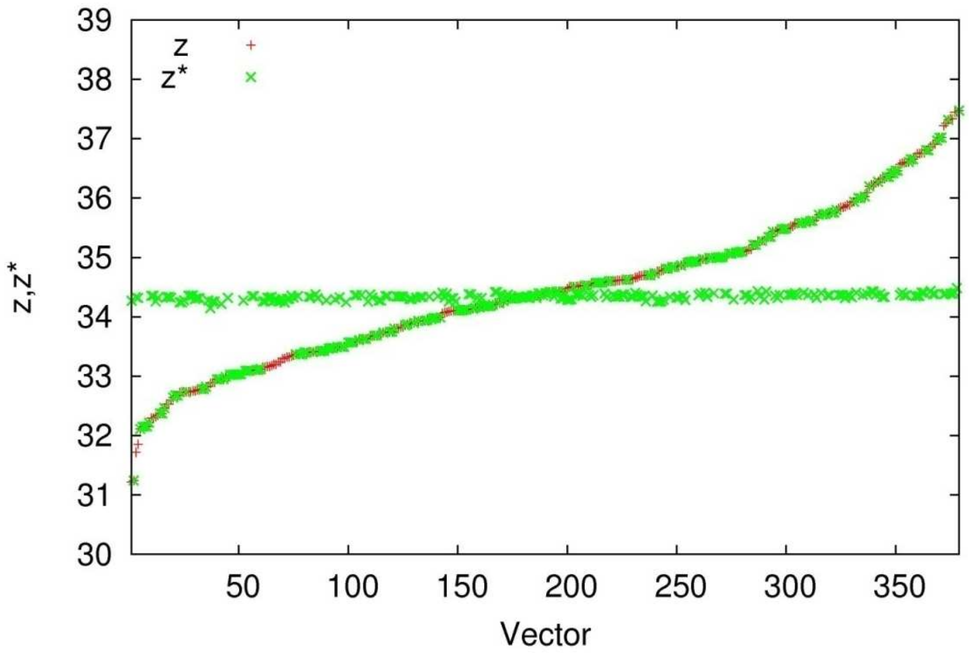
"real world" engineering data (crash test)

- 46 input dimensions in $\underline{x}$
- 1 output in z
- size of dataset: 379
  132 (35%) used for training
- training with: artificial neural network (ANN)
  radial basis function network (RBFN)
  extreme learning machine (ELM)
- requirement: training time < 10 min

Example 1

RBFN

- topology (after optimization): 46–122–1
- errors (absolute):
  - mean: 0.5292
  - sum: 200.58
  - max: 3.0494
  - stand. dev.: 0.55448



good approximation of training data – no for test data

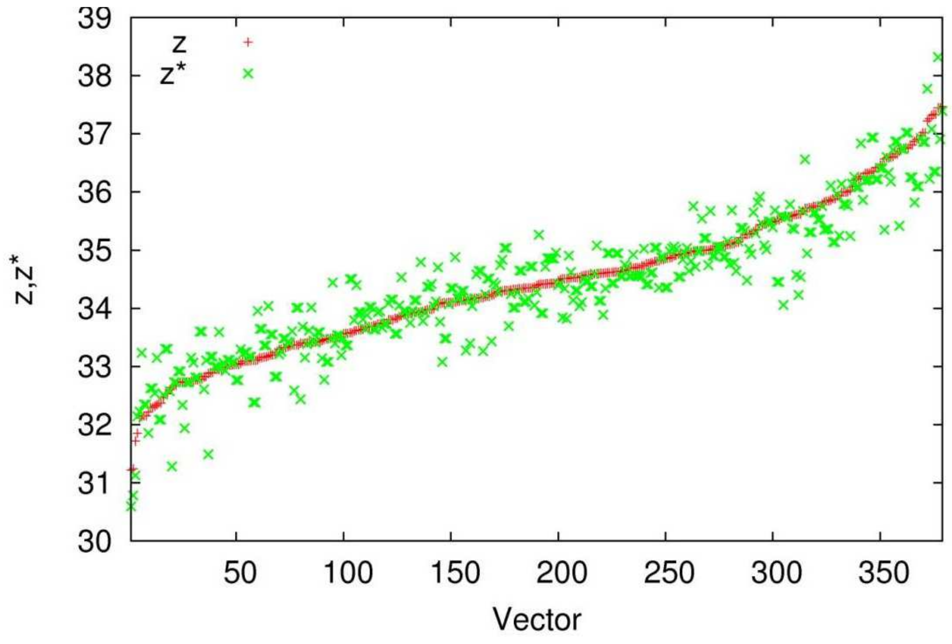
Example 1

topology: 46–95–1

errors (absolute):

- mean: 0.328
- sum: 124.4
- max: 1.518
- stand. dev: 0.088

ELM



trend is approximated

Example 2

damaged RC frame structure

2 floors, 10.8 x 20.4 x 11.1 m^3

- framed in longitudinal direction
- designed against vertical loads

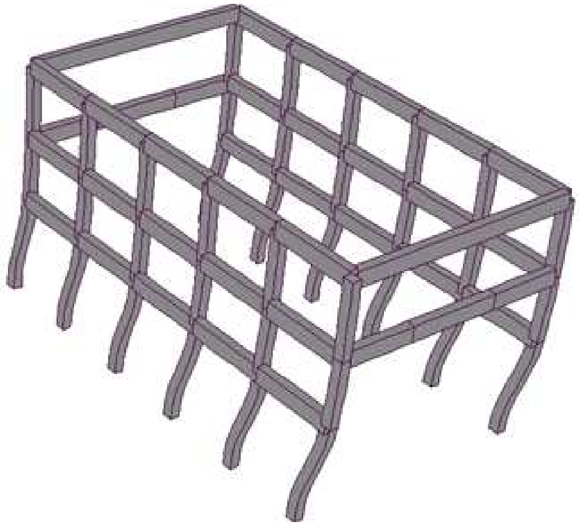
(photos by Alberto Mandara)

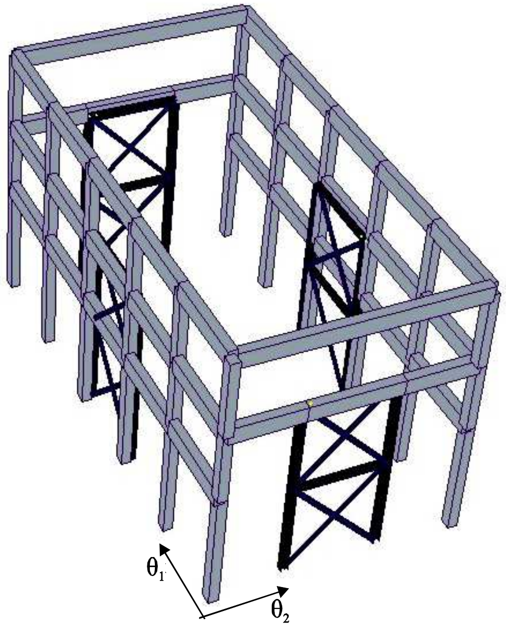
Example 2

vulnerability analysis [Mandara et al. 2009]

2 dominated eigenmodes
mass participation factor > 95%

→ **two independent SDOF system for both directions**





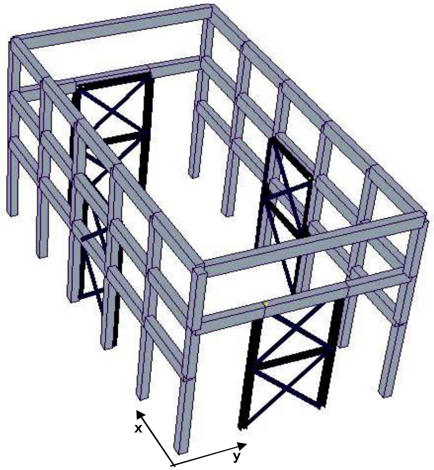
design of bracing system

based on simplified nonlinear 2DOF system

steel braces
with additional energy dissipation devices

connection on the first floor level
purely viscous devices

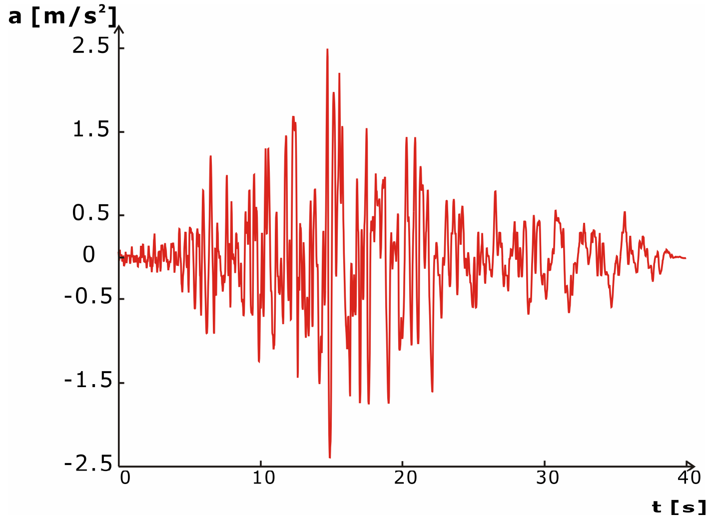
Example 2

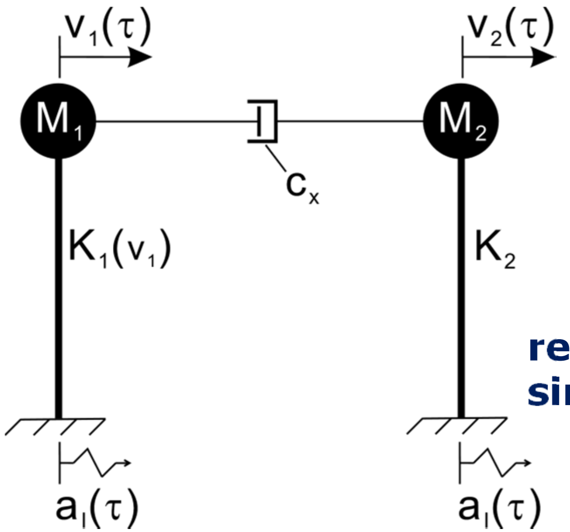


RC structure

Taiwan earthquake 1999

acceleration scaled
to PGA value of 0.25 g



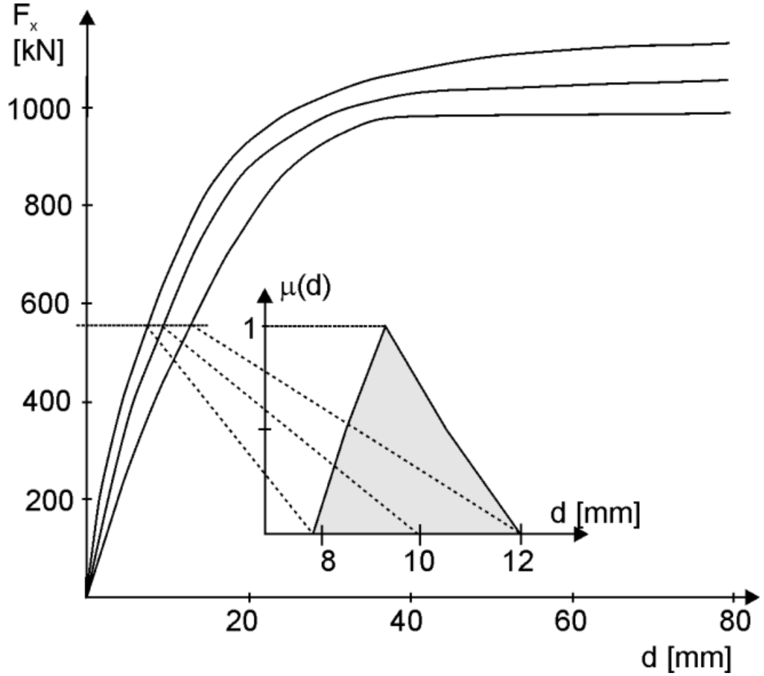


reduced simulation model

Example 2

fuzzy push-over analysis (fuzzy capacity curve)

- compute with the plane bar model



- uncertain **material parameters**

compressive and tensile strength

$\tilde{f}_{ck}$ = <14, 16.5, 20> N/mm²

$\tilde{f}_{t}$ = <1.5, 2.0, 2.5> N/mm²

cross section area of reinforcement

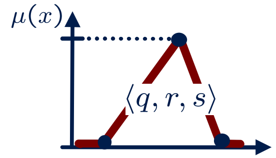
$\tilde{A}$ = <2.69, 3.14, 3.21> cm²

steel braces

fuzzy stiffness $\tilde{K}_2$ = <50, 52.5, 55> MN/m

fuzzy mass $\tilde{M}_2$ = <1.2, 1.5, 1.8> t

connecting device

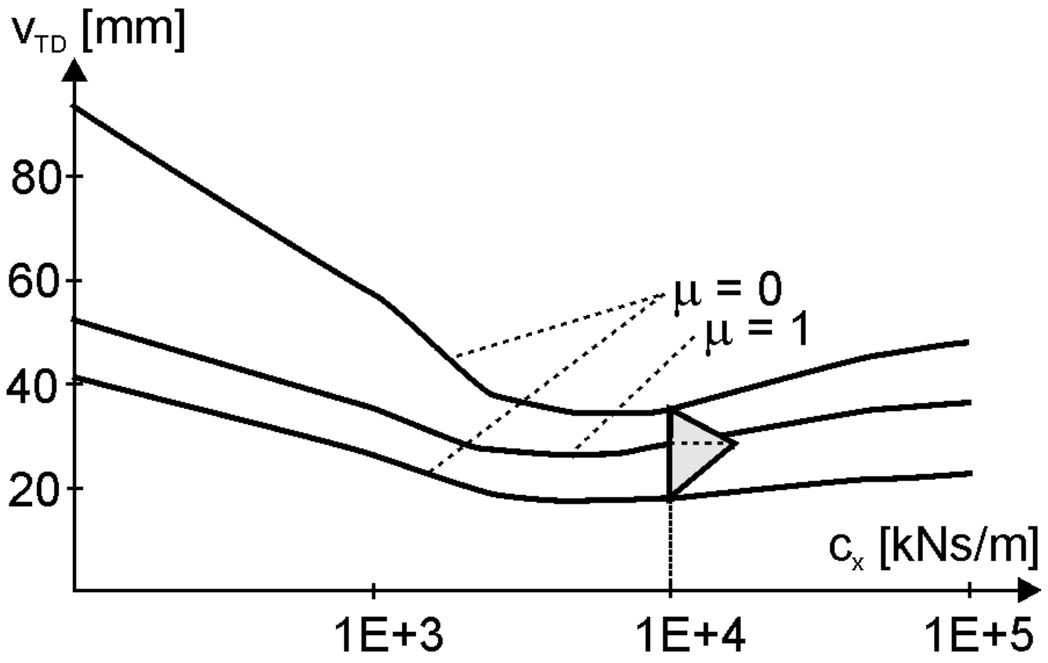


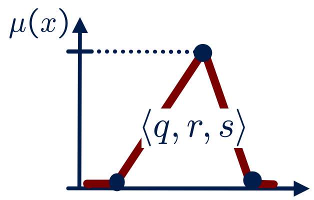
fuzzy viscosity $\tilde{c} = \tilde{b} \cdot c_x$

with $\tilde{b}$ = <0.9, 1.0, 1.1>

Example 2

fuzzy top displacement in dependency of the viscosity

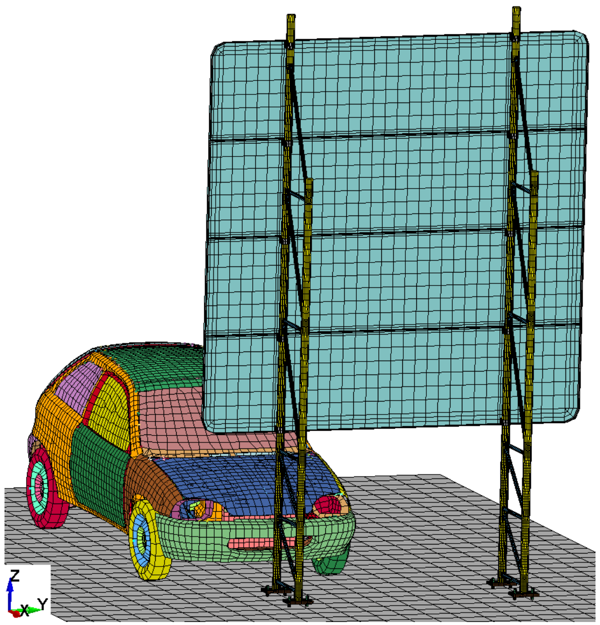




Example 3

car crash with steel girder

- car model (NCAC)
- dummy- and airbag model (DYNAmore)
- crash simulations
  deterministic/uncertain (ISD)
- **optimization with uncertain parameters**
  stiffness steel girder with traffic sign (ISD)
- passive safety (EN12767)



Example 3

- **uncertain parameters**

  car velocity, fuzzy interval $v = <35; 50; 90, 100>$ [km/h]

  impact angle, fuzzy triangular number $\varphi = <45; 45; 90>$ [°]

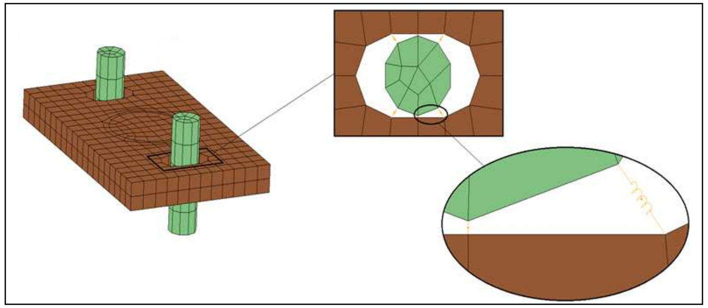
  uncertain part of stiffness of base plate K_d (design value)

  fuzzy random variable for material parameter
  (base plate welded joints)

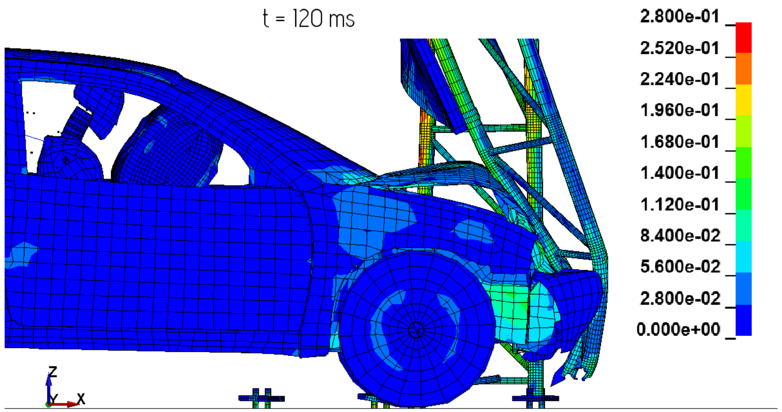
- **fuzzy stochastic analysis**
  (100 simulations)

- **approximation HIC36**
  neural network
  safety relevant parameter

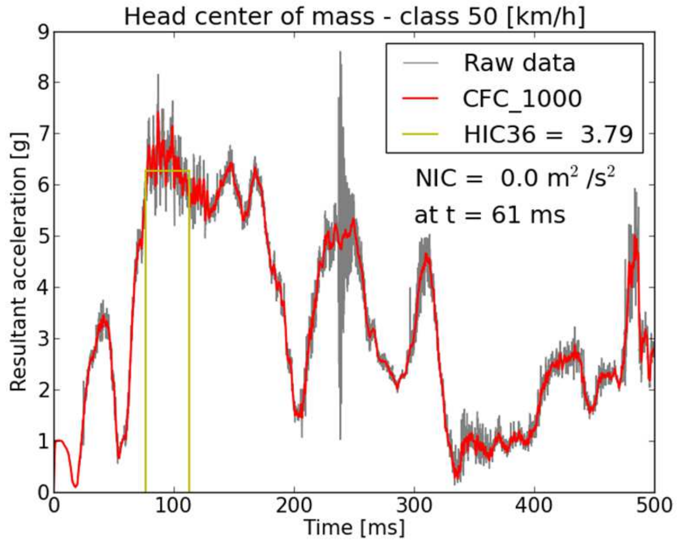
detail base plate



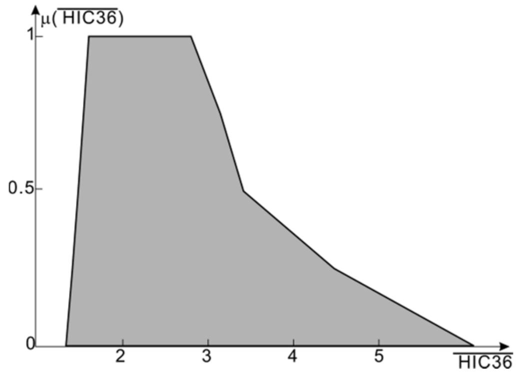
Example 3



von Mises stresses
(v = 50 km/h)



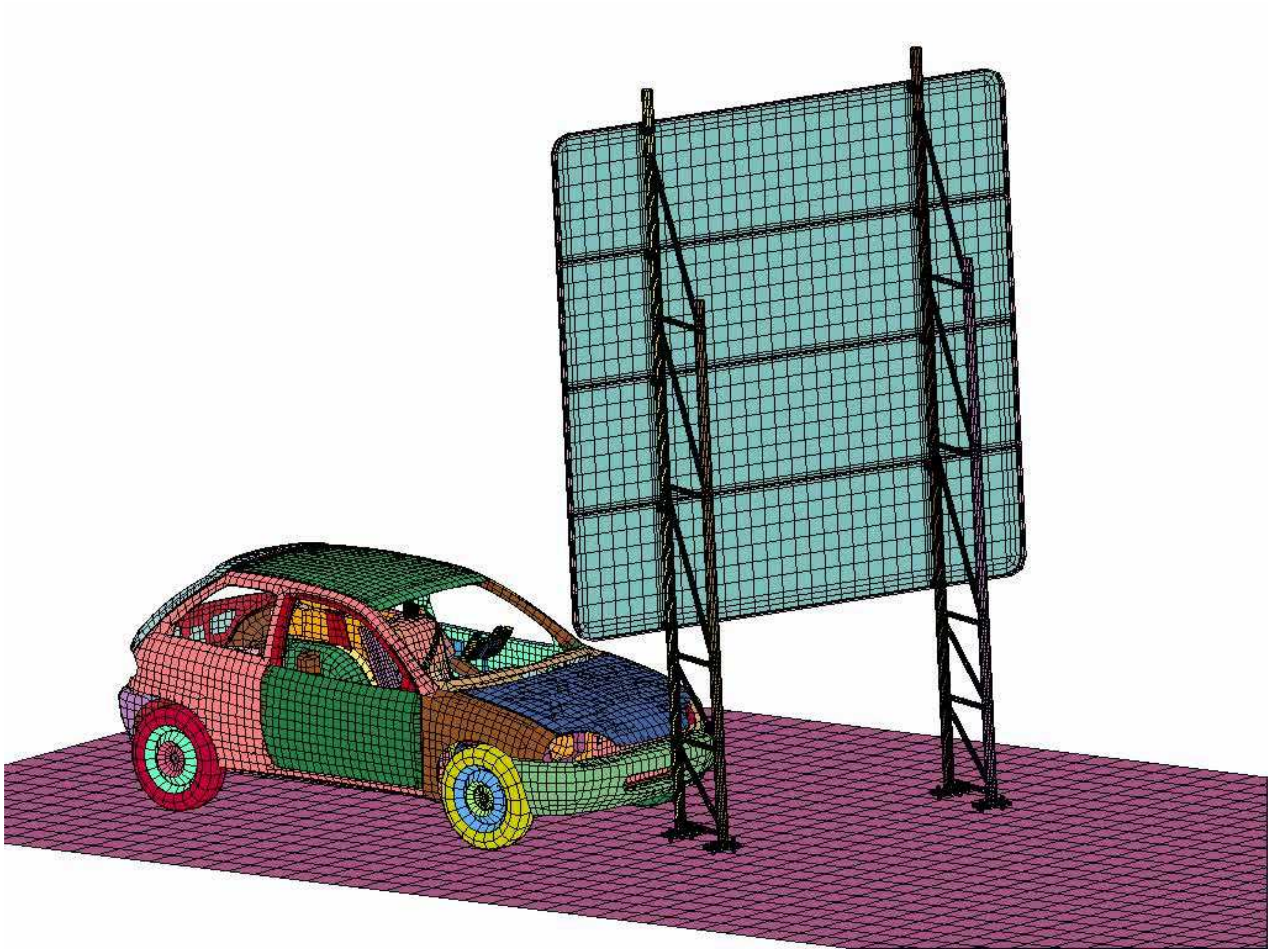
dependency
head acceleration-time



fuzzy mean value HIC36
of optimal design

spring stiffness
optimal design
K_d = 352,3 kN/mm

Example 3



Conclusion

- overview about polymorphic uncertainty models
- structural design tasks with uncertain parameters by formulation and solving of surrogate problems
- comparison of different meta-models
- model reduction
- practical examples

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DRESDEN concept

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