Structural design with polymorphic uncertainty models

Wolfgang Graf
Marco Götz
Michael Kaliske

www.tu-dresden.de/isd
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2 Imprecision and variability
3 Design concepts
   with polymorphic uncertainties
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Introduction

structural analysis and design

- short-term behaviour
  - wind, earthquake
  - impact
  - blasting ...

- long-term behaviour
  - aging and damage
  - creep and fatigue
  - weathering, chemical attack, ...

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problems concerning data uncertainty

• geometry, material and loading data available to a limit extent only
• uncertain observations of geometry, material, and loading data
• varying reproduction conditions during sampling
• information deficit

consequence: polymorphic uncertainty models
## Data uncertainty

<table>
<thead>
<tr>
<th>name</th>
<th>mapping</th>
<th>database characteristics</th>
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</tr>
</thead>
<tbody>
<tr>
<td>random variable</td>
<td>$X : \Omega \rightarrow \mathbb{R}$</td>
<td>deterministic, variability</td>
<td>1 6.16</td>
</tr>
<tr>
<td></td>
<td>$P : \Sigma \rightarrow [0; 1]$</td>
<td></td>
<td>2 5.23</td>
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<td></td>
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<td>3 6.95</td>
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<td>4 6.47</td>
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<td>N 5.85</td>
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</tbody>
</table>

The random variable $f(x)$ can be described by

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

![Random variable distribution graph](image_url)
## Data uncertainty

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<td>aleatoric, a great many</td>
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</tr>
<tr>
<td>fuzzy variable</td>
<td>$\mu_A : \mathbb{R} \rightarrow [0; 1]$</td>
<td>subjective,</td>
<td>epistemic, a few</td>
</tr>
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</table>

**fuzzy variable**

\[
\mu(x) \geq 0 \quad \forall \quad x \in X
\]

\[
\sup_{x \in X} [\mu(x)] = 1
\]
# Data uncertainty

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<td></td>
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<tr>
<td>fuzzy probability based randomness</td>
<td>$X : \Omega \rightarrow \mathbb{R}$</td>
<td>deterministic, a few</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\tilde{P} = (\tilde{P}<em>\beta)</em>{\beta \in (0; 1]}$</td>
<td></td>
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</table>

representation of generalized uncertainty model variables

**fuzzy-random variable**

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-0.5 \left(\frac{x - \tilde{\mu}}{\sigma}\right)^2}$$

<table>
<thead>
<tr>
<th>set of uncertain quantities</th>
<th>e.g. $\mathcal{D}(\Omega, \mathbb{R})$</th>
<th>set of random numbers</th>
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<td>$\mathcal{D}(Q, R)$</td>
<td></td>
<td>$\mathcal{D}(\mathbb{R}, [0; 1])$</td>
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## Polymorphic uncertainty

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</table>
| random variable                           | $X : \Omega \rightarrow \mathbb{R}$  
  $P : \Sigma \rightarrow [0; 1]$ | deterministic, very many | aleatoric (variability) |
| fuzzy value                               | $\mu_A : \mathbb{R} \rightarrow [0; 1]$ | subjective, few        | epistemic               |
| fuzzy random variable                     | $X : \Omega \rightarrow \mathcal{F}(\mathbb{R})$  
  $P : \Sigma \rightarrow [0; 1]$ | imprecise, (very) many | variability, impreciseness |
| fuzzy probability based random variable   | $X : \Omega \rightarrow \mathbb{R}$  
  $\bar{P} = (\bar{P}_\beta)_{\beta \in (0;1)}$ | deterministic, some    | variability, incompleteness |
| fuzzy probability based fuzzy random variable | $X : \Omega \rightarrow \mathcal{F}(\mathbb{R})$  
  $\bar{P} = (\bar{P}_\beta)_{\beta \in (0;1)}$ | imprecise, some        | variability, impreciseness, incompleteness |
Fuzzy probability based fuzzy random variable
Numerical design with uncertain data

structural analysis
\[ \tilde{x}(t) \rightarrow \tilde{z}(t) \]
e.g.
\[ \tilde{M} \cdot \Delta \tilde{v}(\tau) + \tilde{D} \cdot \Delta \tilde{v}(\tau) + \tilde{K}_T \cdot \Delta \tilde{v}(\tau) = \Delta \tilde{F}(\tau) \]

structural design

design parameters, a priori parameters
objective(s), constrains

design methods

- investigation of variants
- optimization (e.g. wait and see, here and now)
- solution of inverse problem (e.g. cluster methods)
- evaluation
Numerical design concepts with uncertain data

Objective function with uncertain quantities

- Deterministic objective function → deterministic optimization task
  \[ f_Z : X \subset \mathbb{R}^n \rightarrow \mathbb{R} \]
  \[ z_{min} = \min_{x \in E} f_Z(x) \]

- Objective function with uncertain quantities
  \[ \hat{f}_Z^u : \mathcal{D}(Q,R) \rightarrow \mathcal{D}(V,W) \]

\[
\begin{align*}
\text{uncertain input quantities} & \\
\text{uncertain design variables} & \\
\text{transformation } \mathcal{T} & \\
\text{deterministic design variables } d & \\
\text{uncertain a priori parameters } p^u
\end{align*}
\]

\[
\begin{align*}
\hat{f}_Z^u : \mathbb{R}^{n_x} \times \mathcal{D}(Q_p, R_p) \rightarrow \mathcal{D}(V,W) : (d,p^u) \mapsto \hat{f}_Z^u(\mathcal{T}(d),p^u)
\end{align*}
\]
General description

• **task**
  
  determine $L \subseteq X_d^+$ such that $\forall \; d \in X_d^+$ and $d_{\text{min}} \in L$ it holds:
  
  $f^u_Z(d_{\text{min}}, p^u) \text{ is } \text{"lower or equal than" } f^u_Z(d, p^u)$
  
  $\implies \min_{d \in X_d^+} f^u_Z(d, p^u)$

• **permissible range**
  
  $X_d^+ = \{ d \in \mathbb{R}^n \mid \forall \; j \in \{1; \ldots; a_g\} : g^u_j(d, p^u) \text{ ,"lower than" } 0$ 
  
  $\land \forall \; k \in \{1; \ldots; a_h\} : h^u_k(d, p^u) \text{ ,"equal" } 0\}$
  
  $g^u_j : \mathcal{D}(Q, R) \to \mathcal{D}(V, W)$
  $h^u_k : \mathcal{D}(Q, R) \to \mathcal{D}(V, W)$

• **surrogate problem**
  
  ➢ **passive approach** (wait-and-see)
  
  determine for all $p$ (realization of $p^u$)
  
  that set $L \subseteq X_d^+$, such that
  
  $\forall \; d \in X_d^+$ and $d_{\text{min}} \in L$ it holds:
  
  $f_Z(d_{\text{min}}, p) \leq f_Z(d, p)$

  ➢ **active approach** (here-and-now)
  
  determine $L \subseteq X_d^+$ such that
  
  $\forall \; d \in X_d^+$ and $d_{\text{min}} \in L$ it holds:
  
  $\mathcal{M}(f^u_Z(d_{\text{min}}, p^u)) \leq \mathcal{M}(f^u_Z(d, p^u))$
  
  $(\mathcal{M} : \mathcal{D}(V, W) \to \mathbb{R})$
Passive and active approach

- **passive approach**
  - minimum of all deterministic objective functions
  - set of design variables
  - range of minimal results

- **active approach**
  - deterministic design variables
  - uncertain result values
  - different design objectives (e.g. reliability, durability, robustness)
Summary of passive and active approach

- **passive approach**
  (wait-and-see)
  - uncertain minimum, uncertain design parameters
  - no uncertain response parameters
  - checking of constraints in post computation
  - fast detection of optimal ranges

- **active approach**
  (here-and-now)
  - deterministic minimum, deterministic design parameters
  - early reduction of information
  - checking of constraints directly
  - expensive detection of the optimum

sequential application

- **early design stage**, application of fuzzy model
- **final design stage**, application of random based models
Numerical frameworks

Passive approach:
- Deterministic optimization
- Stochastic analysis
- Fuzzy analysis
- Uncertain set of designs
- Transformation (design update)
- Structural objective function
- Optimization objective function
- Checking of permissibility

Active approach:
- Deterministic design
- Deterministic optimization
- Preprocessing (reduction of uncertainty)
- Fuzzy analysis
- Stochastic analysis
- Structural objective function
- Optimization objective function
- Postprocessing
- Checking of permissibility
## Surrogate models and reduction methods

| **model reduction** | condensation techniques  
|                     | compression techniques  
|                     | substructure techniques |
| **sensitivity analysis** | derivate/variance based  
|                           | global/local |
| **parallelization** |  |
| **surrogate models** | response surface methods (meta-models)  
|                       | „intelligent“ approaches |
| **„model free“ approach** | computational intelligence  
|                           | artificial neural networks |
Principal functionality of meta-models

![Diagram of a 3D response surface approximation with input parameters and response values.]

4-step procedure to build a metamodel

- **Design of experiments (DOE)**
  - e.g. grid-like, random
- **Surrogate model**
  - e.g. polynomial, ANN
- **Model fitting**
  - e.g. regression, training
- **Model validation**

RSA ... response surface approximation
Example 1

“real world” engineering data (crash test)

- 46 input dimensions in $x$
- 1 output in $z$
- size of dataset: 379
  - 132 (35%) used for training
- training with: artificial neural network (ANN)
  radial basis function network (RBFN)
  extreme learning machine (ELM)
- requirement: training time < 10 min
Example 1

- topology (after optimization): 46–122–1
- errors (absolute):
  - mean: 0.5292
  - sum: 200.58
  - max: 3.0494
  - stand. dev.: 0.55448

good approximation of training data – no for test data
Example 1

topology: 46–95–1

errors (absolute):
• mean: 0.328
• sum: 124.4
• max: 1.518
• stand. dev: 0.088

trend is approximated
Example 2

damaged RC frame structure

2 floors, 10.8 x 20.4 x 11.1 m³

• framed in longitudinal direction
• designed against vertical loads

(photos by Alberto Mandara)
Example 2

vulnerability analysis [Mandara et al. 2009]

2 dominated eigenmodes
mass participation factor > 95%

→ two independent SDOF system
for both directions

design of bracing system
based on simplified nonlinear 2DOF system
steel braces
with additional energy dissipation devices
connection on the first floor level
purely viscous devices
Example 2

**RC structure**

**Taiwan earthquake 1999**
acceleration scaled to PGA value of 0.25 g

**Reduced simulation model**

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Example 2

**fuzzy push-over analysis (fuzzy capacity curve)**
- compute with the plane bar model
  - uncertain **material parameters**
    - compressive and tensile strength
      \[ \tilde{f}_{ck} = <14, 16.5, 20> \text{ N/mm}^2 \]
      \[ \tilde{f}_t = <1.5, 2.0, 2.5> \text{ N/mm}^2 \]
    - cross section area of reinforcement
      \[ \tilde{A} = <2.69, 3.14, 3.21> \text{ cm}^2 \]

**steel braces**
- fuzzy stiffness \( \tilde{K}_2 = <50, 52.5, 55> \text{ MN/m} \)
- fuzzy mass \( \tilde{M}_2 = <1.2, 1.5, 1.8> \text{ t} \)

**connecting device**
- fuzzy viscosity \( \tilde{\varsigma} = \tilde{b} \cdot \varsigma_x \)
  - with \( \tilde{b} = <0.9, 1.0, 1.1> \)
Example 2

fuzzy top displacement in dependency of the viscosity

![Diagram showing the fuzzy top displacement in dependency of the viscosity with plots for different values of mu (0 and 1) against c_x [kNs/m] and v_{TD} [mm].]
Example 3

car crash with steel girder

• car model (NCAC)
• dummy- and airbag model (DYNAmore)
• crash simulations
deterministic/uncertain (ISD)
• **optimization with uncertain parameters**
stiffness steel girder with traffic sign (ISD)
• passive safety (EN12767)
Example 3

- **uncertain parameters**
  
  car velocity, fuzzy interval \( v = <35; 50; 90, 100> \) [km/h]
  
  impact angle, fuzzy triangular number \( \varphi = <45; 45; 90> \) [°]
  
  uncertain part of stiffness of base plate \( K_d \) (design value)
  
  fuzzy random variable for material parameter (base plate welded joints)

- **fuzzy stochastic analysis**
  (100 simulations)

- **approximation HIC36**
  neural network
  safety relevant parameter
Example 3

von Mises stresses
($v = 50 \text{ km/h}$)

dependency
head acceleration-time

dependency

fuzzy mean value HIC36
of optimal design

spring stiffness
optimal design
$K_d = 352.3 \text{ kN/mm}$
Example 3
Conclusion

- overview about polymorphic uncertainty models
- structural design tasks with uncertain parameters by formulation and solving of surrogate problems
- comparison of different meta-models
- model reduction
- practical examples
Structural design
with polymorphic uncertainty models

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