Surrogate modeling for mechanized tunneling simulations with uncertain data

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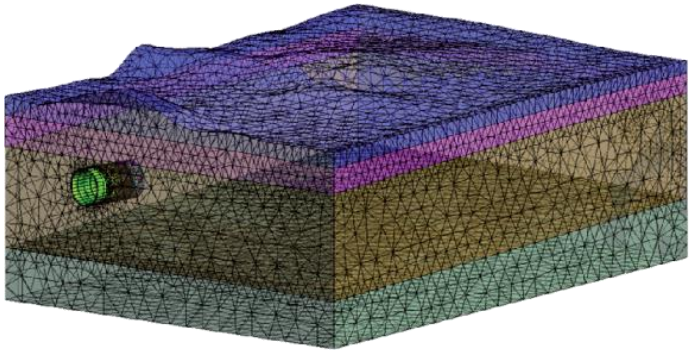
Content

1. Motivation
2. Process-oriented FE model for mechanized tunneling
3. Numerical simulation with polymorphic uncertain data
4. Surrogate modeling for uncertain processes
5. Examples
6. Conclusion / Outlook

Computational models

advanced finite element simulation

- a priori – tunnel design
- during construction – steering



Uncertainties

limited information

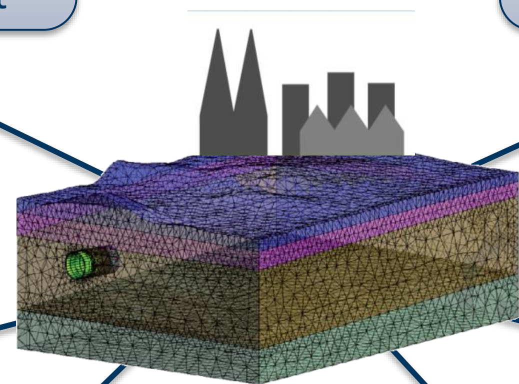
- geometry and heterogeneity of soil layers
- construction process

} uncertain parameters

Process-oriented simulation model – subsystems

Subsystem I:
geology,
geotechnical model

Subsystem II:
face support



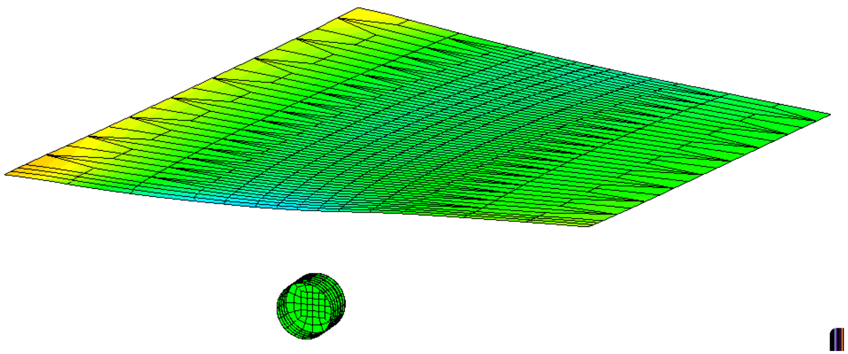
Subsystem III:
soil excavation

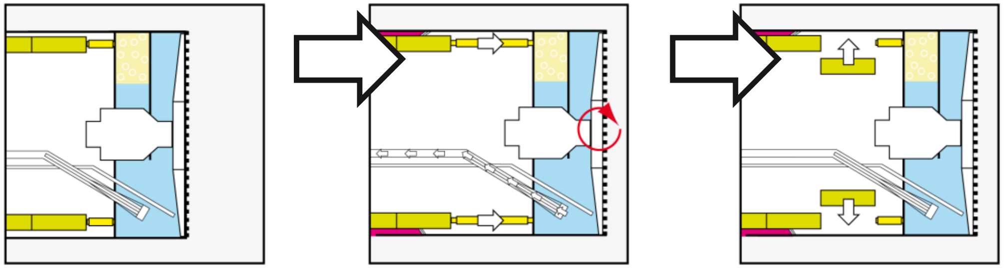
Subsystem IV:
tunnel boring machine,
steering

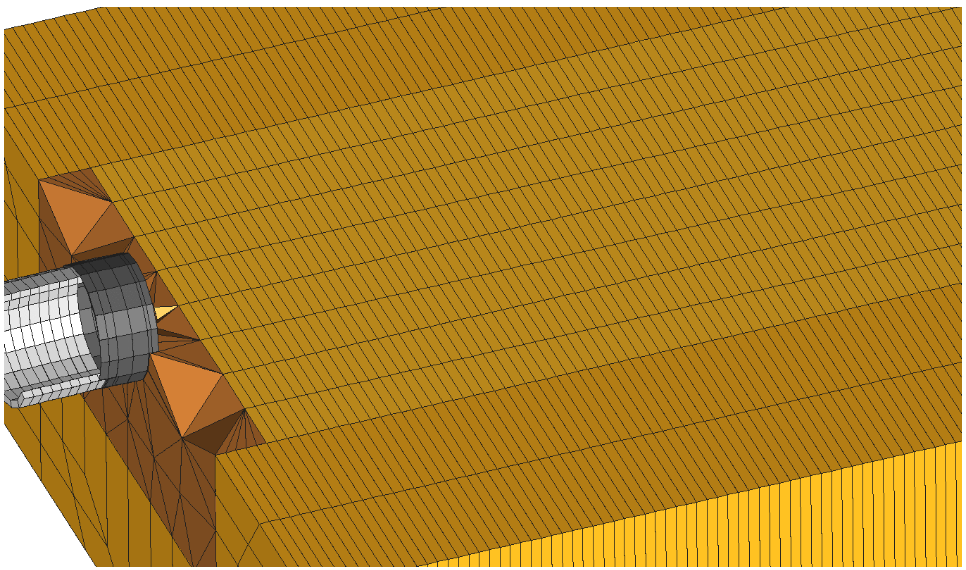
Subsystem VI:
existing buildings

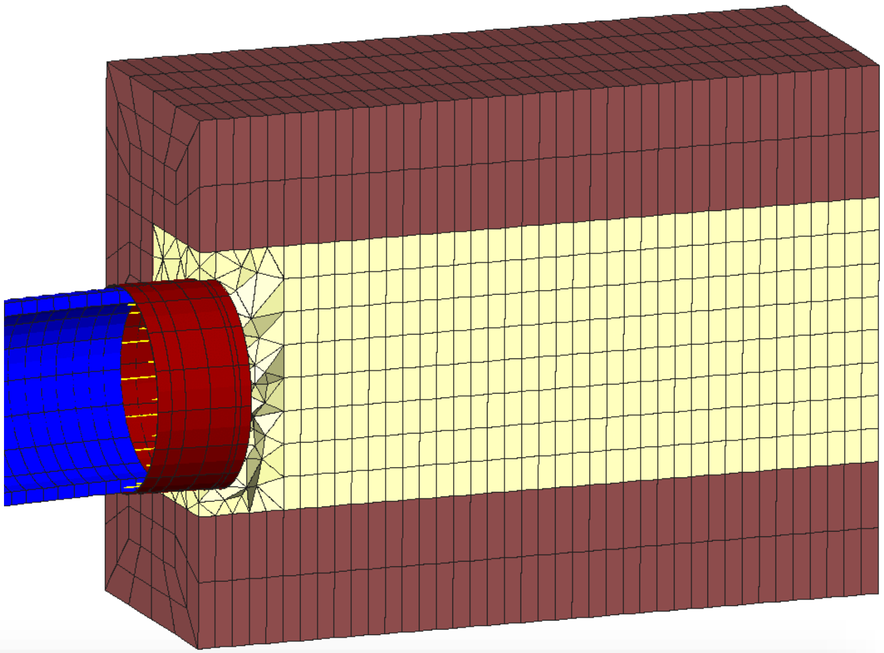
Subsystem V:
lining

Process-oriented simulation model – steering

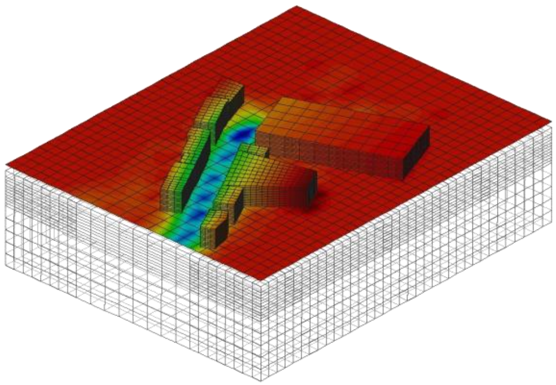




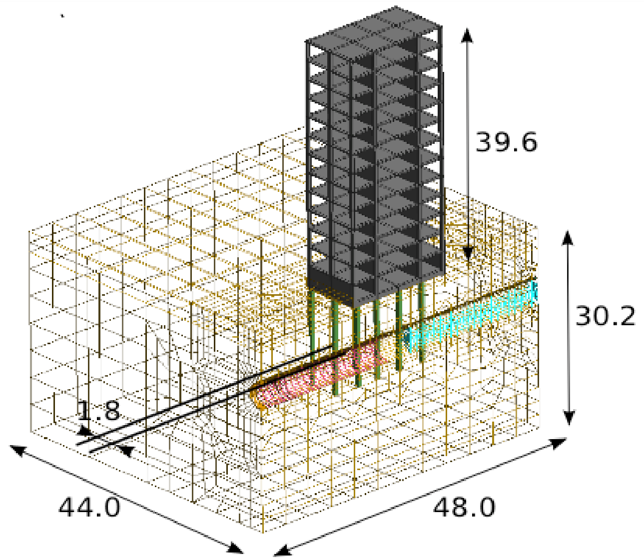


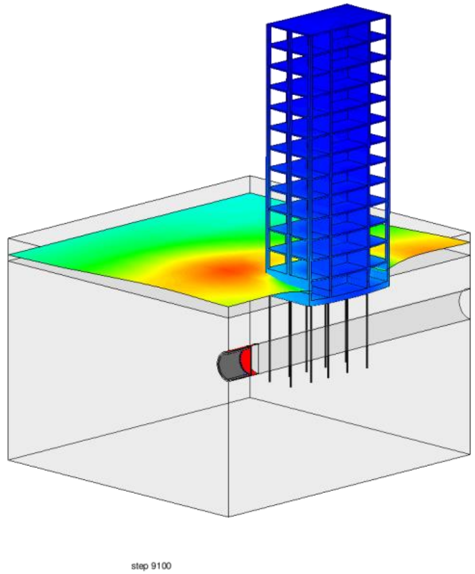


Process-oriented simulation model – interaction with buildings



surrogate models for buildings

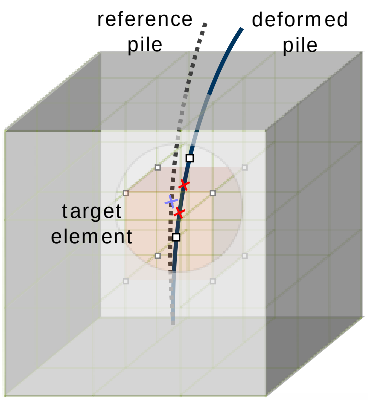




element for pile foundation

Embedded beam element

- independent of the soil mesh
- arbitrary number and orientation per element
- skin friction, pile tip resistance



J. Ninic, J. Stascheit & G. Meschke, Int. J. Num. Anal. Meth. Geomechanics 2013

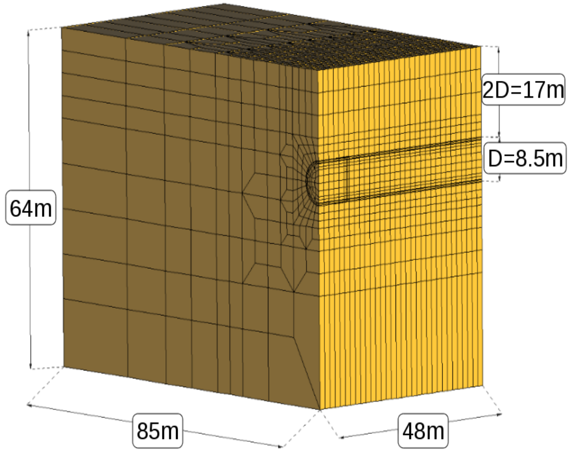
Numerical structural analysis with uncertain data

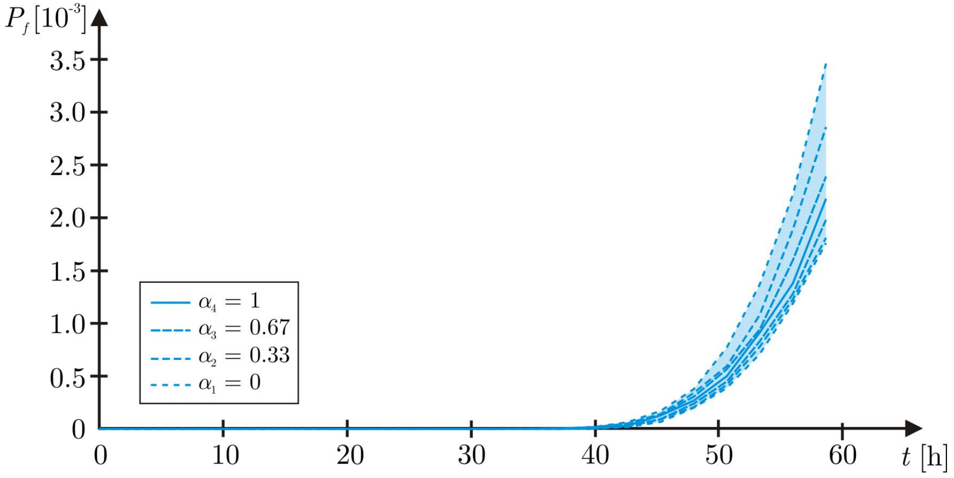
modeling

loading
geometry
material behavior

numerical analysis

uncertain structural responses
structural reliability





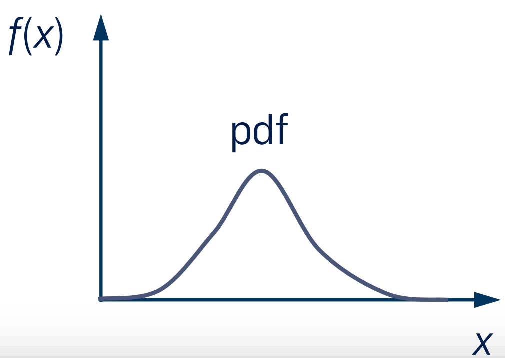
Simulation with uncertain data

uncertainty

aleatory

- randomness / variability
- not reducible
- objective assessment

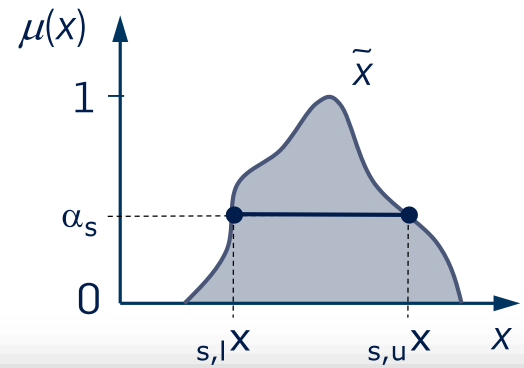
stochastic numbers



epistemic

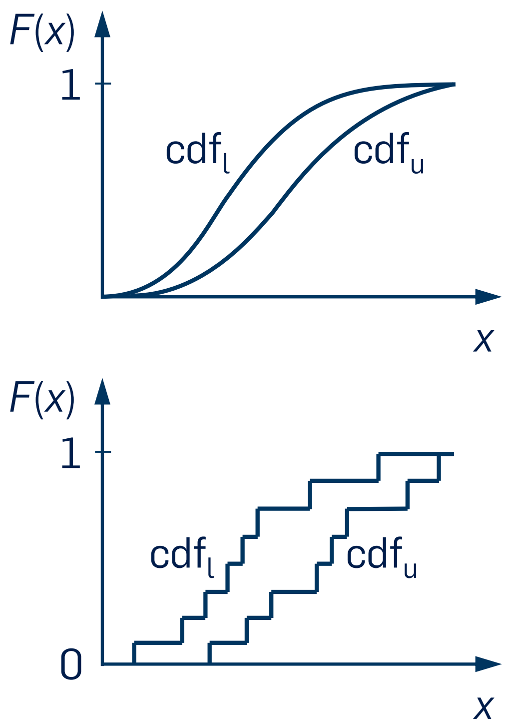
- lack of knowledge
- reducible
- subjective / objective assessment

intervals, fuzzy numbers

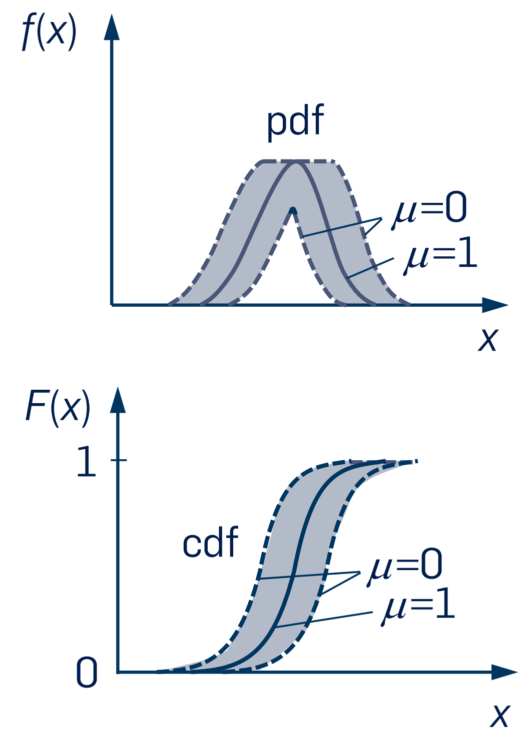


Generalized uncertainty models

probability boxes



fuzzy stochastic numbers



Stochastic analysis

inputs

stochastic
numbers

$X \longrightarrow Z$

outputs

stochastic
structural responses

stochastic analysis

deterministic analysis

Interval analysis

inputs			outputs
intervals	$\bar{x}$ → $\bar{z}$		interval structural responses

- interval arithmetic
- optimization

interval analysis

deterministic analysis

Fuzzy analysis

inputs			outputs
fuzzy numbers	$\tilde{x}$ → $\tilde{z}$		fuzzy structural responses

- fuzzy arithmetic
- α-level optimization

fuzzy analysis

deterministic analysis

Fuzzy stochastic analysis

inputs

stochastic
fuzzy stochastic
interval stochastic
fuzzy numbers
intervals

$\tilde{X} \longrightarrow \tilde{Z}$

outputs

fuzzy stochastic
structural responses

fuzzy analysis

stochastic analysis

deterministic simulation

- surrogate models for deterministic simulation

Simulation with uncertain data

Time-dependent behavior

- process simulation with time constant uncertain inputs

inputs → **outputs**

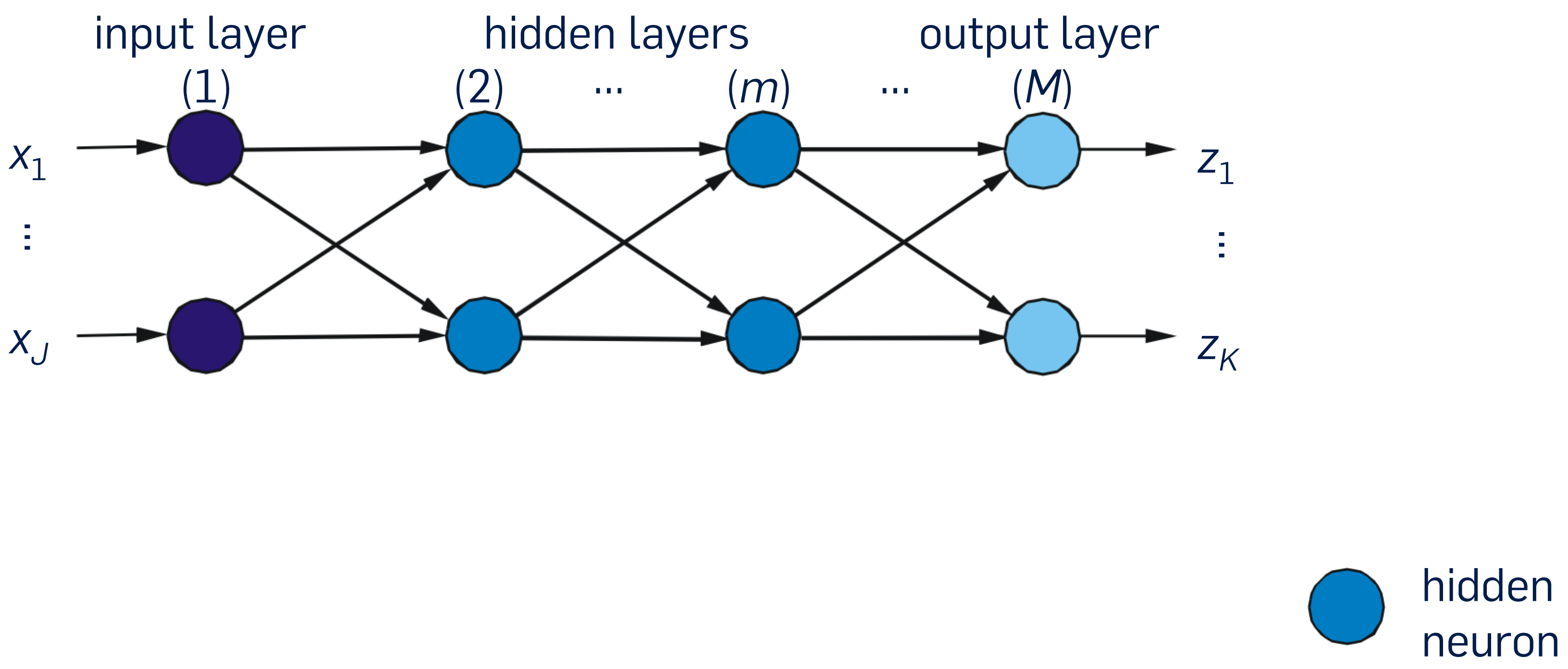
$$\tilde{X} \longrightarrow \tilde{Z}(t)$$

- process simulation with time varying uncertain inputs

inputs → **outputs**

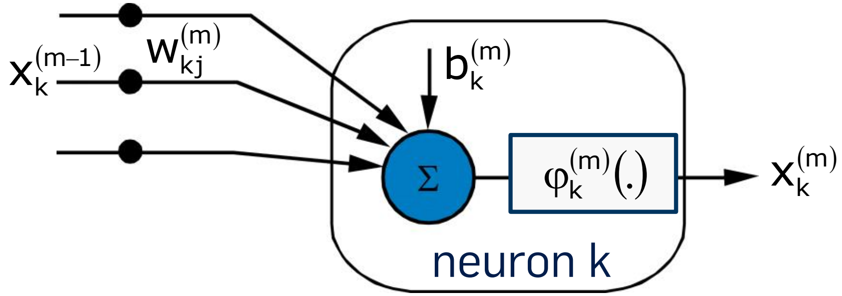
$$\tilde{X}(t) \longrightarrow \tilde{Z}(t)$$

Feed forward neural network – deterministic simulation



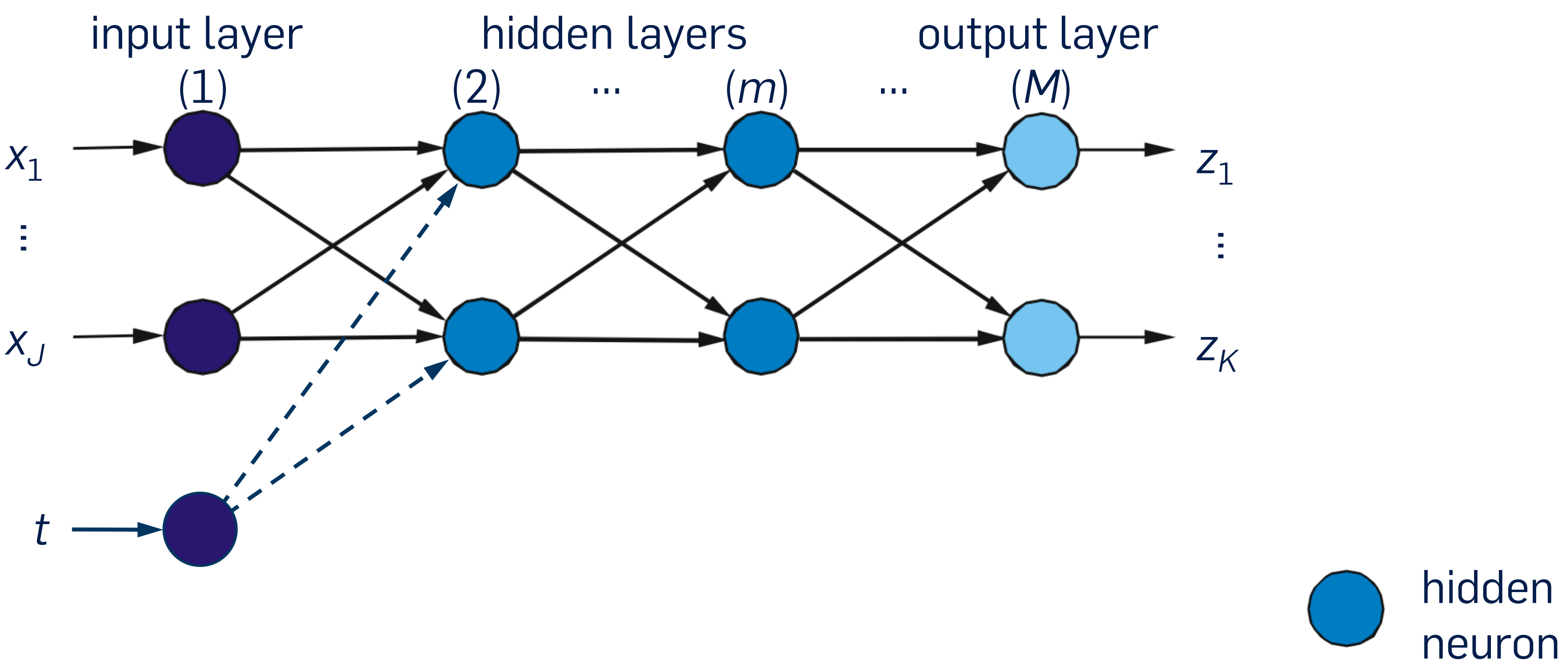
Feed forward neural network – deterministic simulation

- hidden neuron

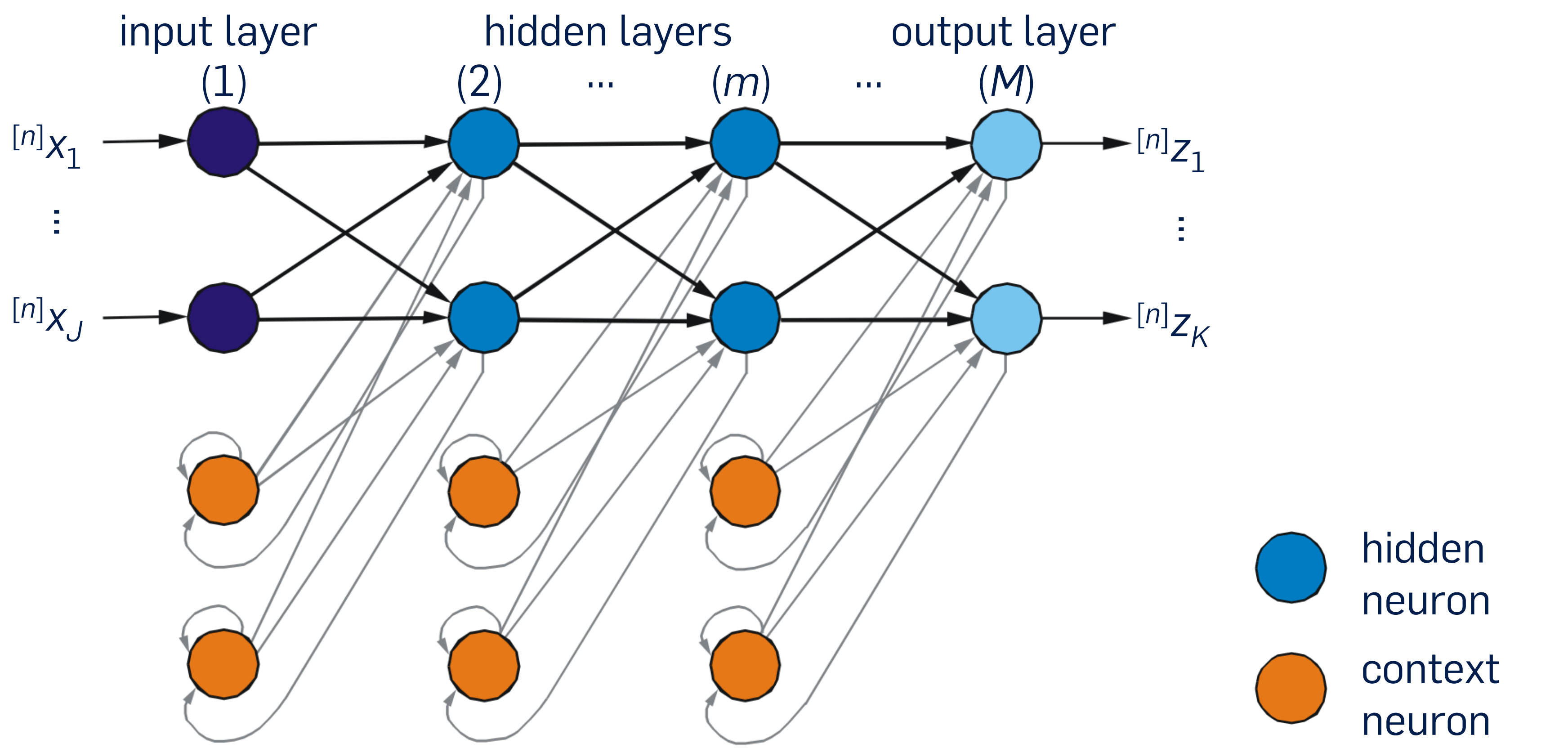


$$x_k^{(m)} = \varphi_k^{(m)}\left(\sum_{j=1}^{J^{(m-1)}}\left[x_j^{(m-1)} \cdot w_{kj}^{(m)}\right] + b_k^{(m)}\right)$$

Feed forward neural network – deterministic simulation

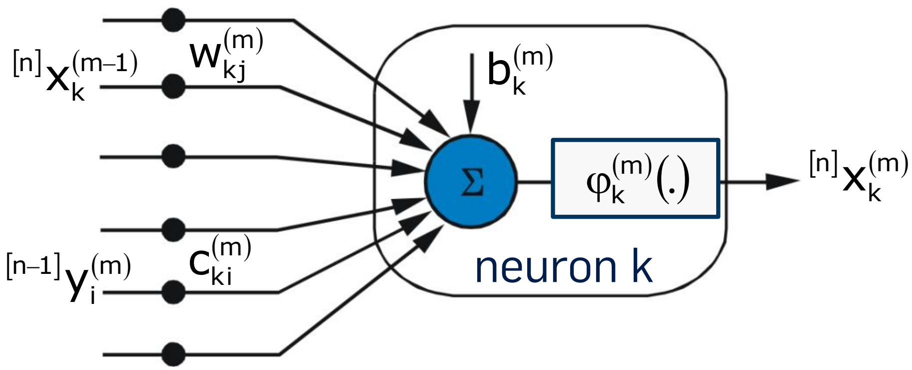


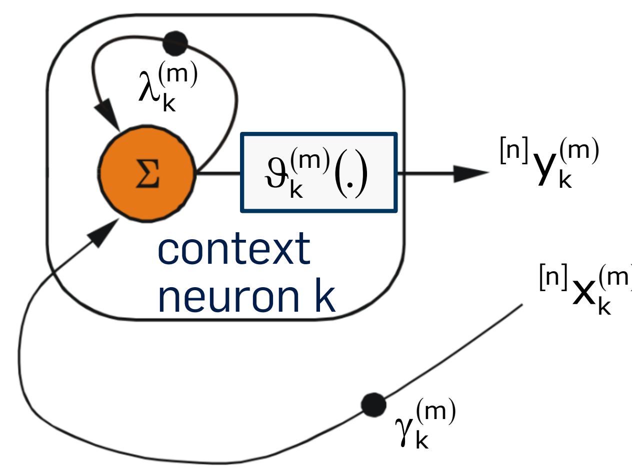
Recurrent neural network – deterministic simulation



Recurrent neural network – deterministic simulation

- hidden neuron
- context neuron



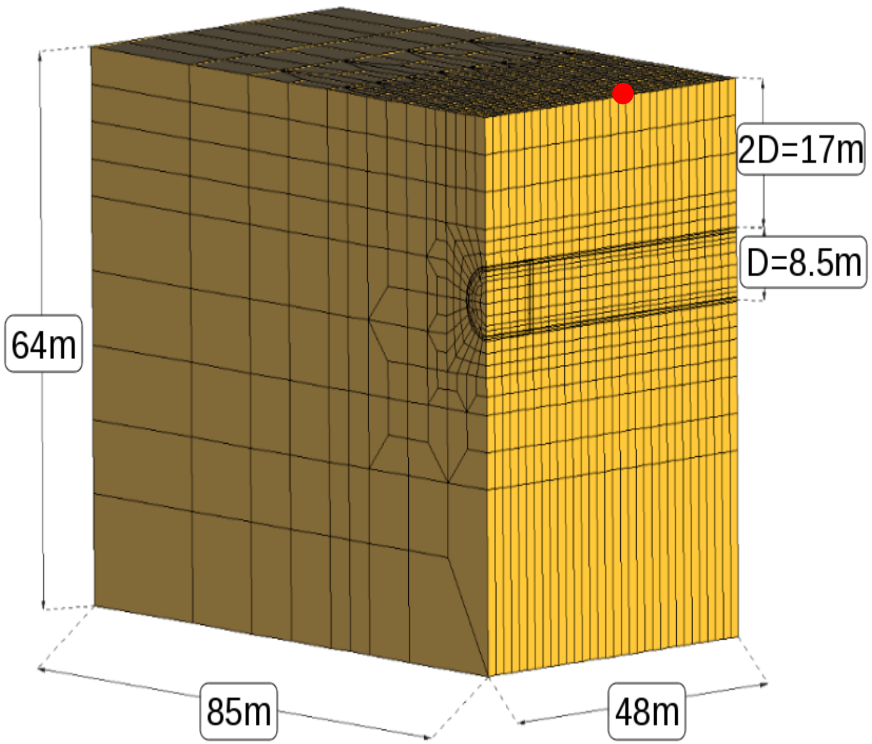


$$^{[n]}x_k^{(m)} = \varphi_k^{(m)}\left(\sum_{j=1}^{J^{(m-1)}} \left[{}^{[n]}x_j^{(m-1)} \cdot w_{kj}^{(m)} \right] + b_k^{(m)} + \sum_{i=1}^{I^{(m)}} \left[{}^{[n-1]}y_i^{(m)} \cdot c_{ki}^{(m)} \right] \right)$$

$$^{[n]}y_k^{(m)} = \vartheta_k^{(m)}\left({}^{[n]}x_k^{(m)} \cdot \gamma_k^{(m)} + {}^{[n-1]}y_k^{(m)} \cdot \lambda_k^{(m)} \right)$$

Example – RNN surrogate model (fuzzy stochastic analysis)

Simulation model



Soil behaviour:
Drucker-Prager yield surface
and isotropic hardening

Poisson ratio ν	0.3
cohesion *c* (kPa)	100
hardening modulus *H* (MPa)	8

Investigated parameters	**Min**	**Max**
Young modulus *E* (MPa)	10	100
internal friction angle ϕ (°)	30	40

Example – RNN surrogate model (fuzzy stochastic analysis)

Neural network based surrogate model

recurrent neural network

- network architecture 2 – 8 – 1

$$\begin{matrix} E \\ \phi \end{matrix} \longrightarrow v(t)$$

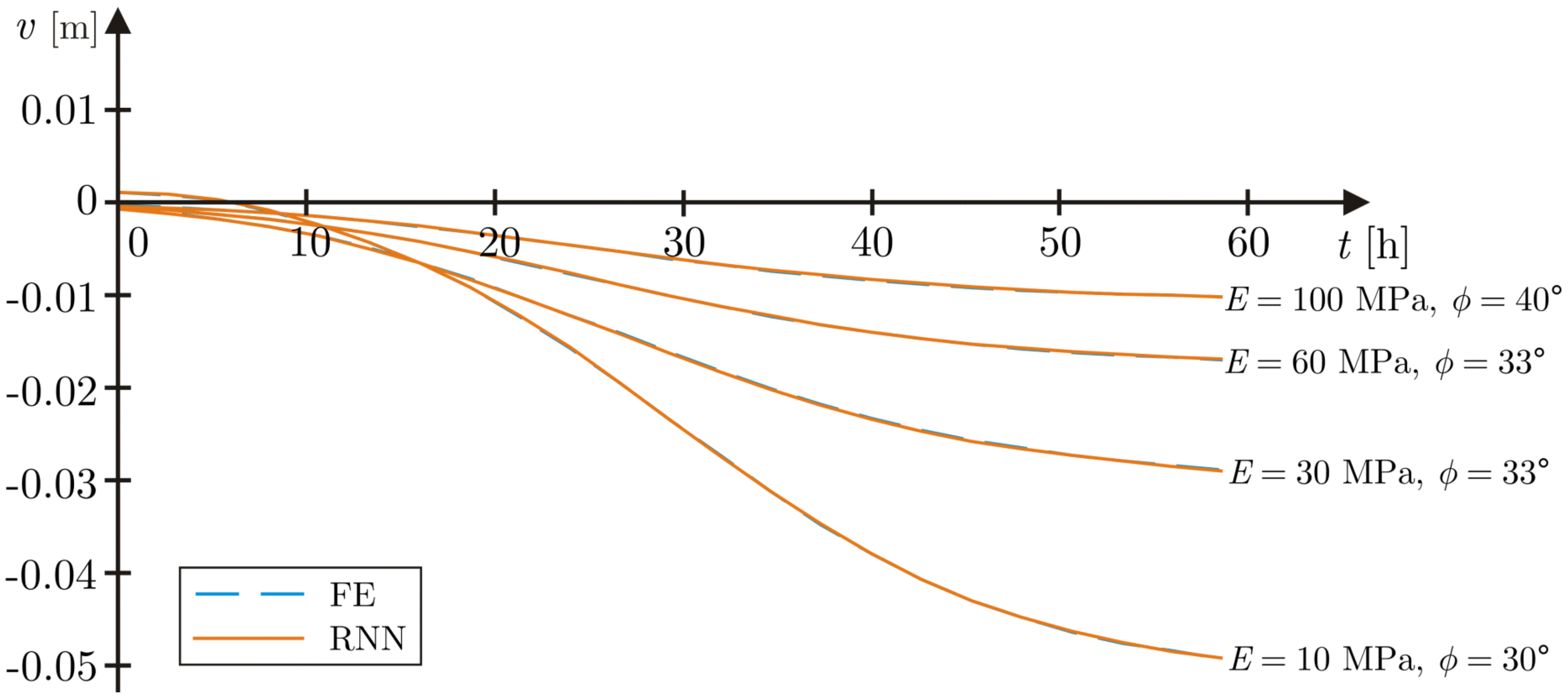
- 9 context neurons for history dependent computation

network training and validation

- 50 training, 50 validation patterns with 22 time steps
- modified backpropagation algorithm 10^7 runs

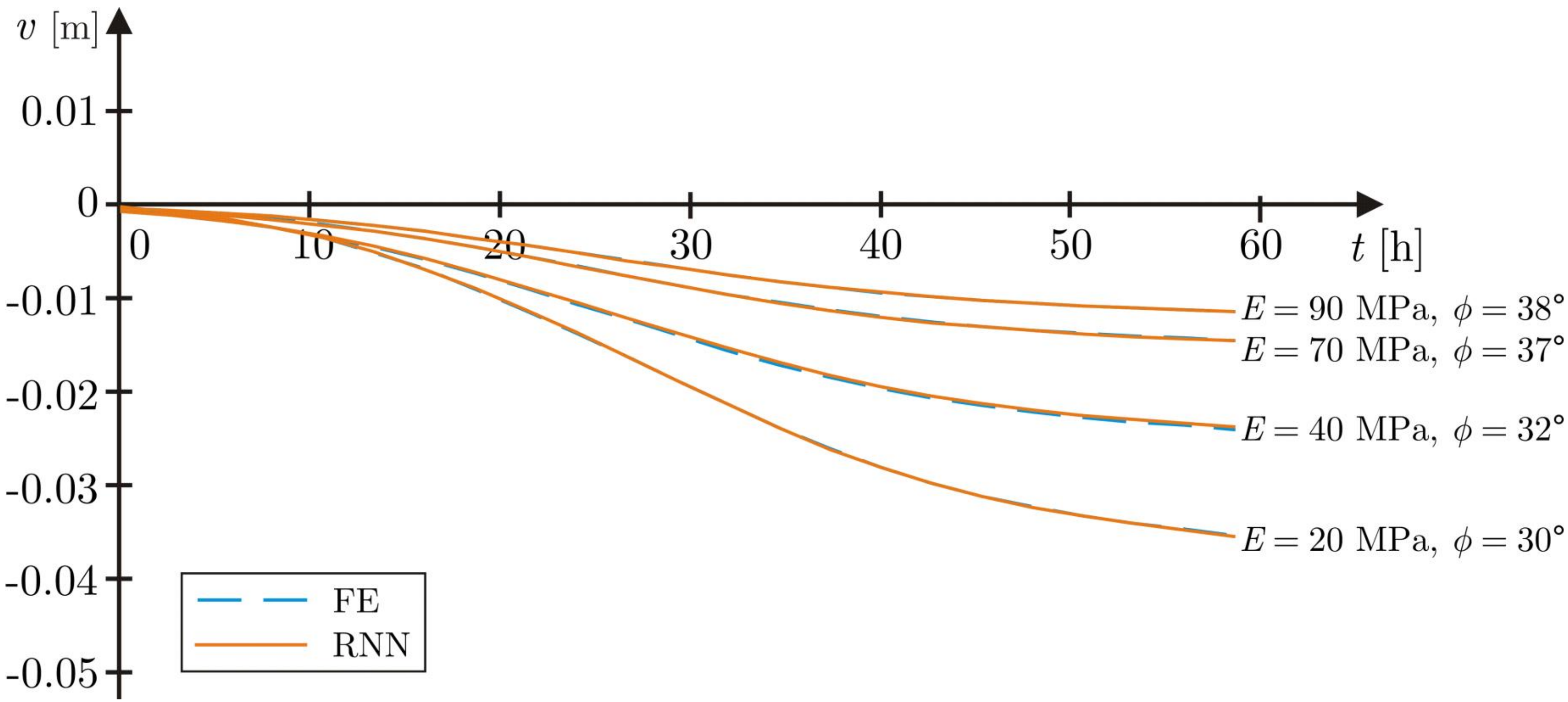
Example – RNN surrogate model (fuzzy stochastic analysis)

Surrogate model – training results



Example – RNN surrogate model (fuzzy stochastic analysis)

Surrogate model – validation results



Example – RNN surrogate model (fuzzy stochastic analysis)

Reliability analysis

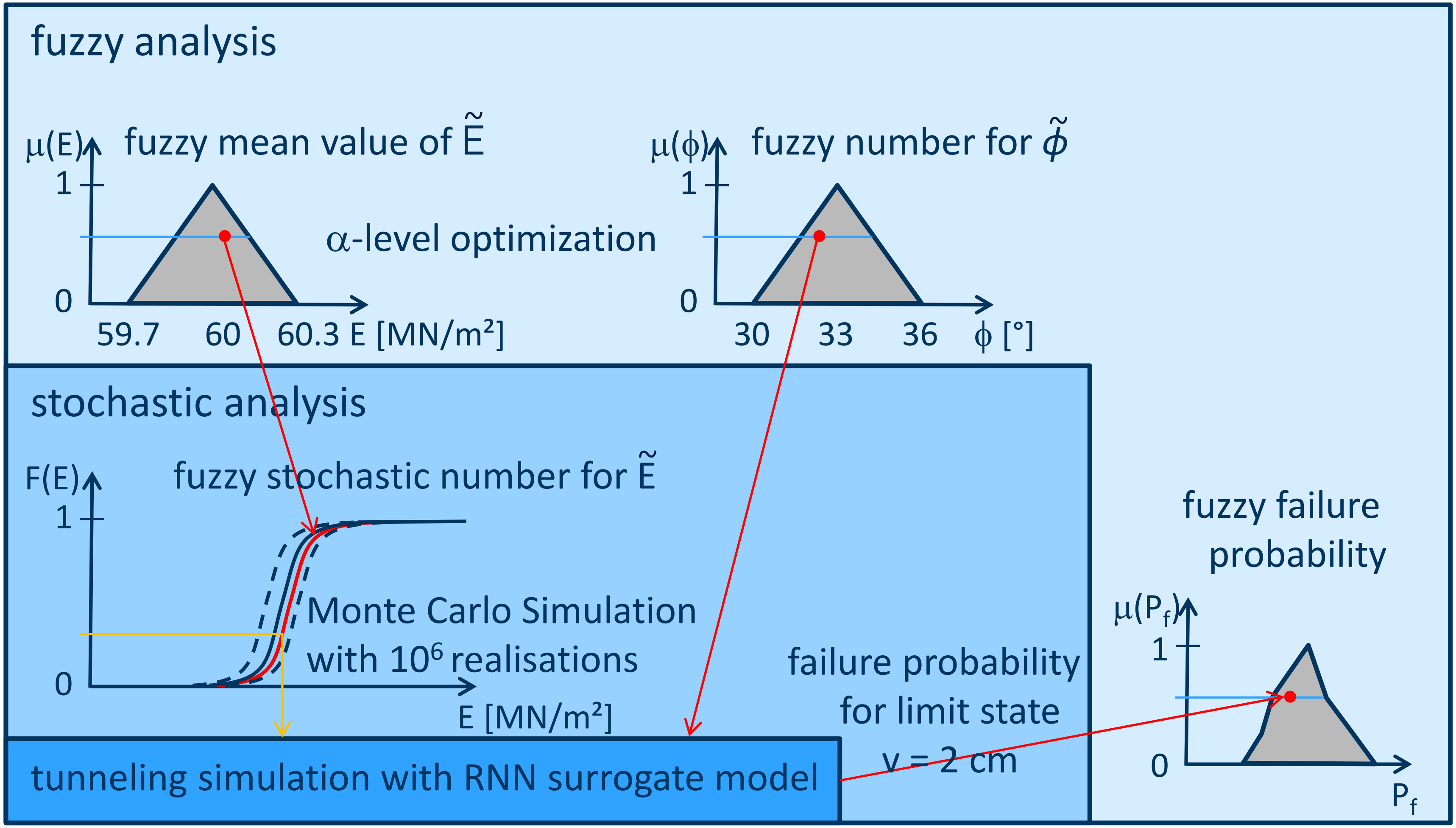
uncertain soil material parameters

- fuzzy stochastic number for modulus of elasticity E
- fuzzy number for internal friction angle ϕ

fuzzy stochastic analysis for each time step

- settlement limit state of 2 cm
- α-level optimization for lower and upper bound P_f
- Monte Carlo simulation with 10^6 samples

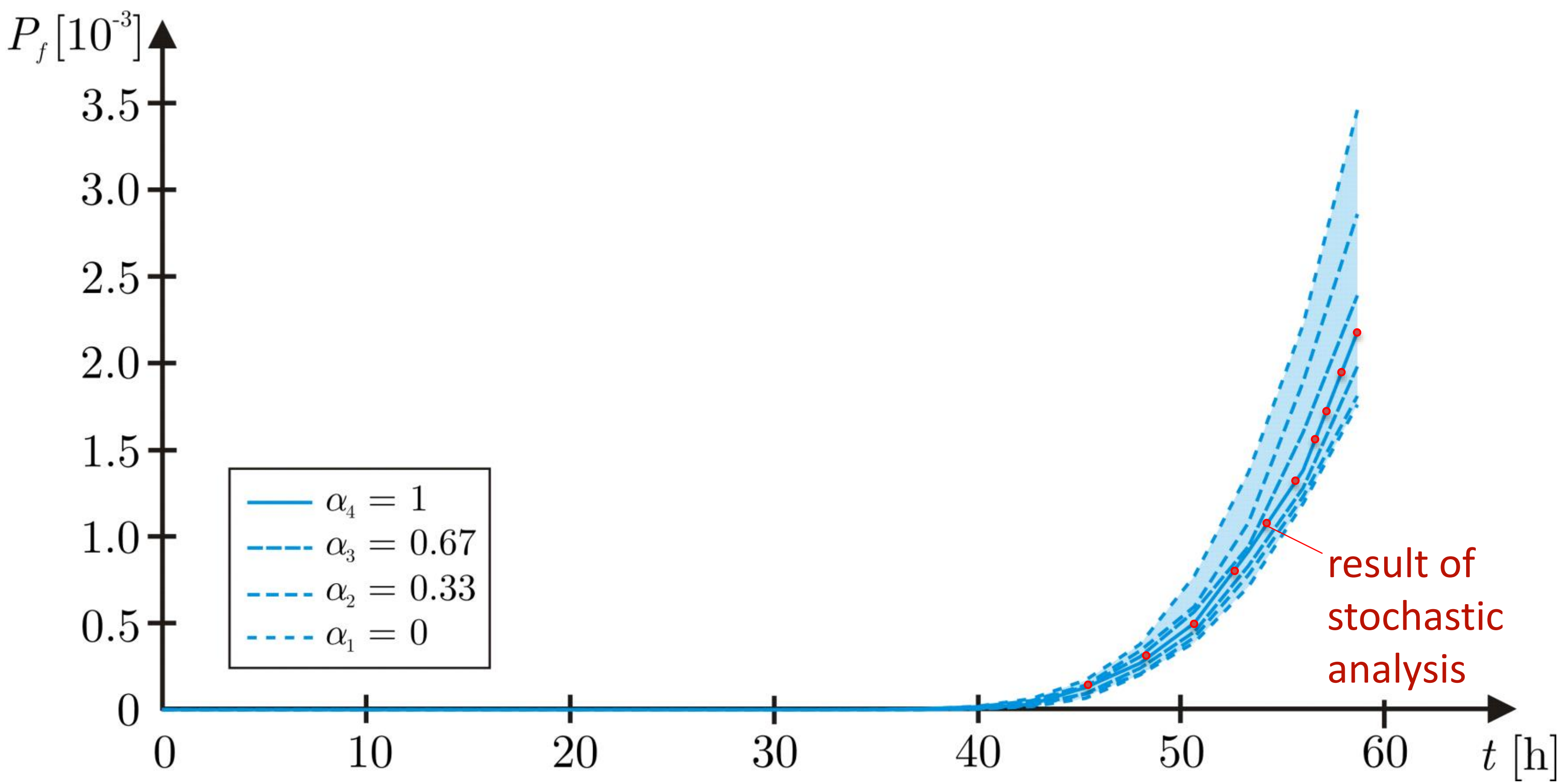
Example – RNN surrogate model (fuzzy stochastic analysis)



B. Möller und M. Beer. Fuzzy Randomness, 2004

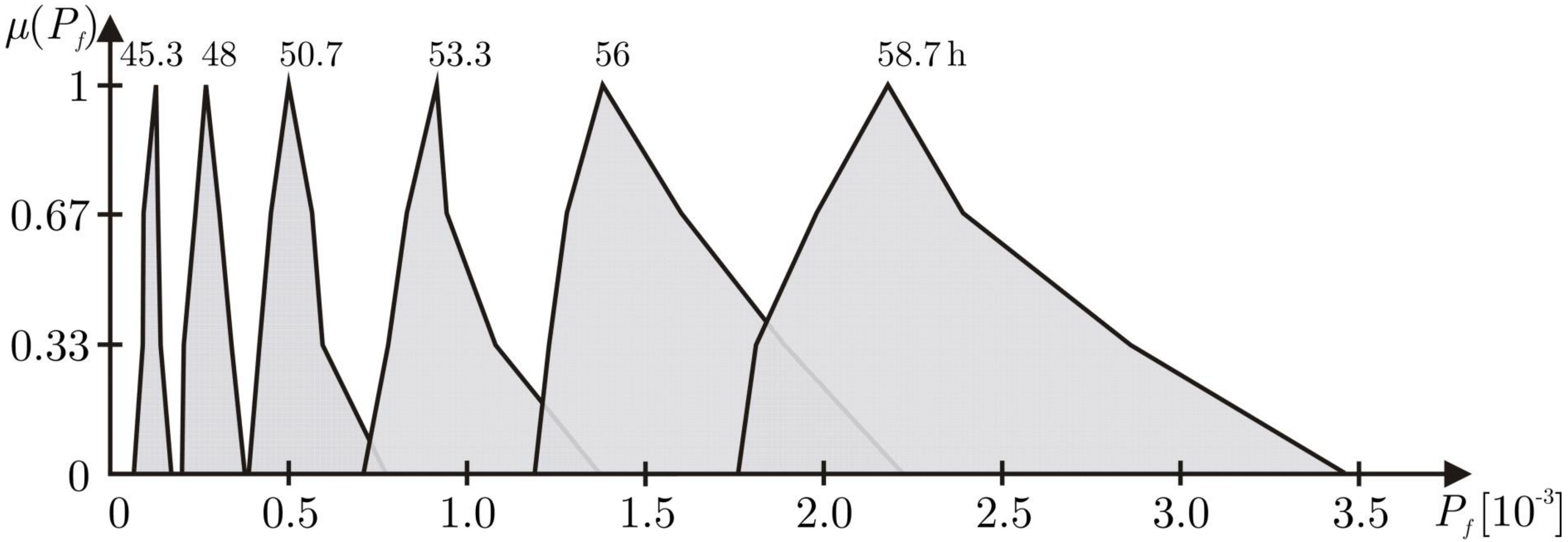
Example – RNN surrogate model (fuzzy stochastic analysis)

Time varying fuzzy failure probability – trajectories

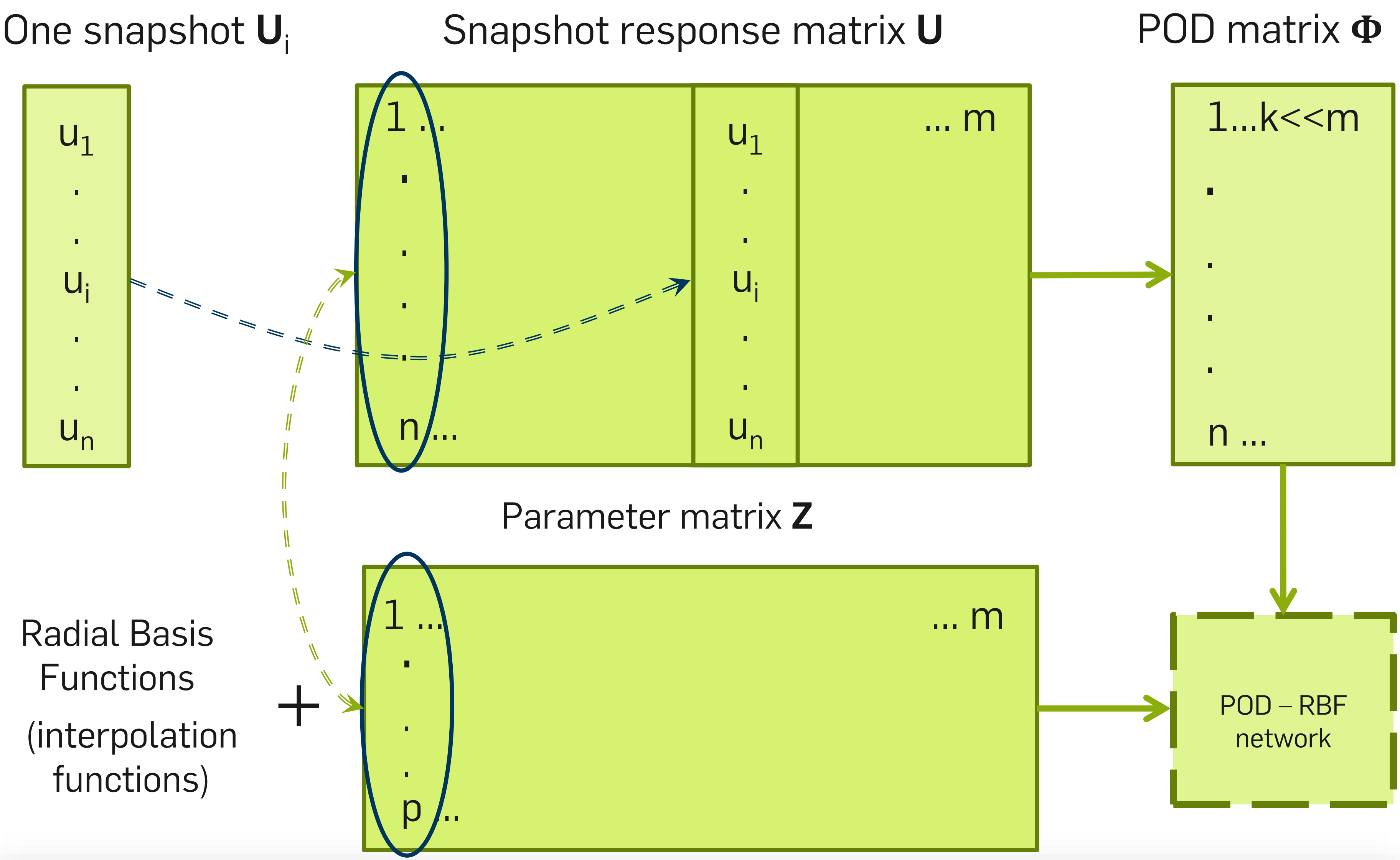


Example – RNN surrogate model (fuzzy stochastic analysis)

Time varying fuzzy failure probability – memberships



Proper Orthogonal Decomposition



Proper Orthogonal Decomposition

Mathematical point of view:

Given set of functions $\{\boldsymbol{u_k} \in \boldsymbol{L^2}(\boldsymbol{\Omega}) \mid \boldsymbol{k} = \mathbf{1}, \dots, \boldsymbol{N}\}$ -> Exists eigenfunctions $\{\boldsymbol{\phi_i}\}_{\boldsymbol{i=1}}^{\infty}$

$$u_k = \sum_{i=1}^{\infty} A_i^k \phi_i$$

Eigenvalue $\boldsymbol{\lambda_i}$ is the average energy in the projection of the ensemble onto the basis function ϕ_i. **Total energy** is given by $\sum_{\boldsymbol{i=1}}^{\infty} \boldsymbol{\lambda_i}$.

Select the **first n POD modes** to get the approximation

$$u_k \cong \tilde{u}_k = \sum_{j=1}^{n} A_i^k \phi_i$$

<u>**POD**</u>: Method for computing the optimal linear basis (modes) for representing a sample set of data (snapshots)

<u>**Goal**</u>: Construct the set of orthonormal vectors $\boldsymbol{\Phi}$ (POD modes, POD basis vectors)

Proper Orthogonal Decomposition

POD can be performed by Singular Value Decomposition (SVD) or solving eigenvalue problem. Given matrix A:

- Compute SVD $A = U \, \Sigma \, V^T$

Basis vector: $\phi^i = U_{.,i} \; and \; \lambda_i = \Sigma_{ii}^2$

- Compute eigenvalue of $AA^T \cdot V = \Lambda \cdot V$

Basis vector: $\phi^i = V_{.,i} \; and \; \lambda_i = \Lambda_{ii}$

- Compute eigenvalue of $A^T A \cdot V = \Lambda \cdot V$

Basis vector: $\phi^i = A \cdot V_{.,i} / \sqrt{\lambda_i} \; and \; \lambda_i = \Lambda_{ii}$

Algorithm: POD procedure to find the basis vectors capture desired accuracy

Input: Snapshots matrix $\mathbf{U}$, desired accuracy E. Output: Truncated POD basis vectors $\bar{\mathbf{\Phi}}$

- Compute covariance matrix $\mathbf{C} = \mathbf{U}^{\mathbf{T}} \cdot \mathbf{U}$
- Compute eigenvalue decomposition $[\mathbf{\Psi}, \mathbf{\Lambda}] = eig(\mathbf{C})$
- Set $\mathbf{\Phi}_i = \mathbf{U} \cdot \mathbf{\Psi}_{-,i} / \sqrt{\lambda_i}$
- Set $\lambda_i = \mathbf{\Lambda}_{ii}$ for $i = 1, \ldots, M$
- Define K based on desired accuracy E as: $\sum_{i=1}^{K} \lambda_i / \sum_{i=1}^{M} \lambda_i \geq \mathrm{E}$
- Return: truncated POD basis $\bar{\mathbf{\Phi}}$ by taking the first K columns of $\mathbf{\Phi}$

Proper Orthogonal Decomposition – Radial Basis Functions

Consider the truncated POD basis vectors $\bar{\boldsymbol{\Phi}}$ of snapshots matrix **U**.

U and a single snapshot of **U** can be approximated as:

$$\mathbf{U} \approx \bar{\boldsymbol{\Phi}} \cdot \bar{\mathbf{A}}$$

$$\mathbf{U}_{\boldsymbol{i}} \approx \bar{\boldsymbol{\Phi}} \cdot \overline{\mathbf{A}_{\boldsymbol{i}}}$$

$\bar{\mathbf{A}}$ and $\overline{\mathbf{A}_{\boldsymbol{i}}}$ are the truncated amplitude matrix and vector, respectively. (constant)

Continuous response: Introduce $\bar{\mathbf{A}}$ and $\overline{\mathbf{A}_{\boldsymbol{i}}}$ as nonlinear function of the vector of input parameters **Z.**

$$\overline{\mathbf{A}_{\boldsymbol{i}}} = \mathbf{B} \cdot \boldsymbol{F}_i$$

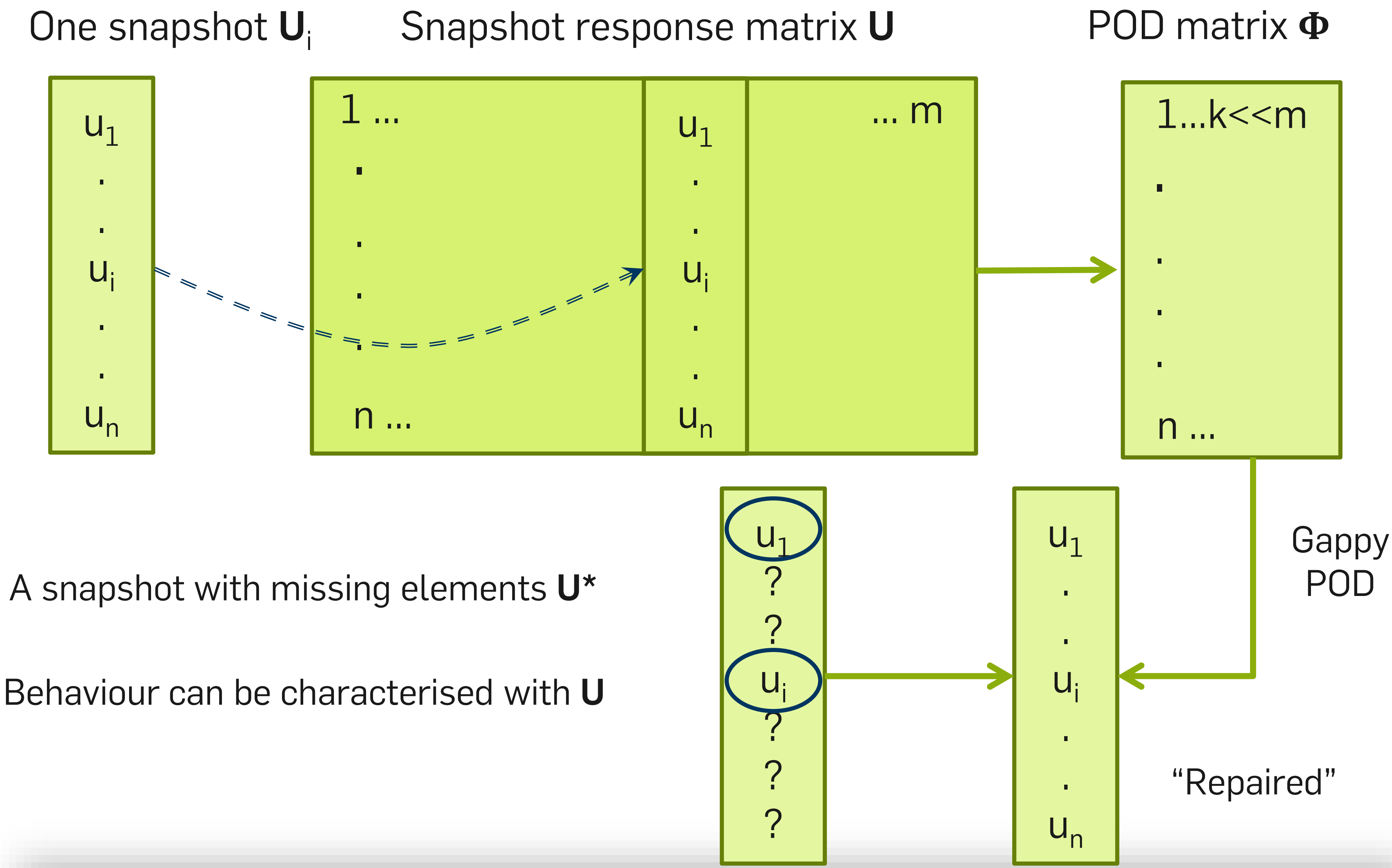
$$\boldsymbol{F}_i = \left[f_1(\boldsymbol{Z}^j) \dots f_j(\boldsymbol{Z}^j) \dots f_M(\boldsymbol{Z}^j)\right]^T$$

$f_j(\boldsymbol{Z})$**:** radial basis functions as interpolation functions

Finally, an approximation of the output with an arbitrary set of input parameters:

$$\mathbf{U}_{\boldsymbol{i}} \approx \bar{\boldsymbol{\Phi}} \cdot \mathbf{B} \cdot \boldsymbol{F}_i$$

Gappy Proper Orthogonal Decomposition



Gappy Proper Orthogonal Decomposition

The repaired vector $\mathbf{U}^{**}$ can be approximated as:

$$\mathbf{U}^{**} = \overline{\mathbf{\Phi}} \cdot \overline{\mathbf{A}^{*}}$$

To compute coefficients $\mathbf{A}^{*}$ the error E between original and repair vectors must be minimized

$$\mathrm{E} = \|\mathbf{U}^{*} - \mathbf{U}^{**}\|^{2}$$

Only available data are compared

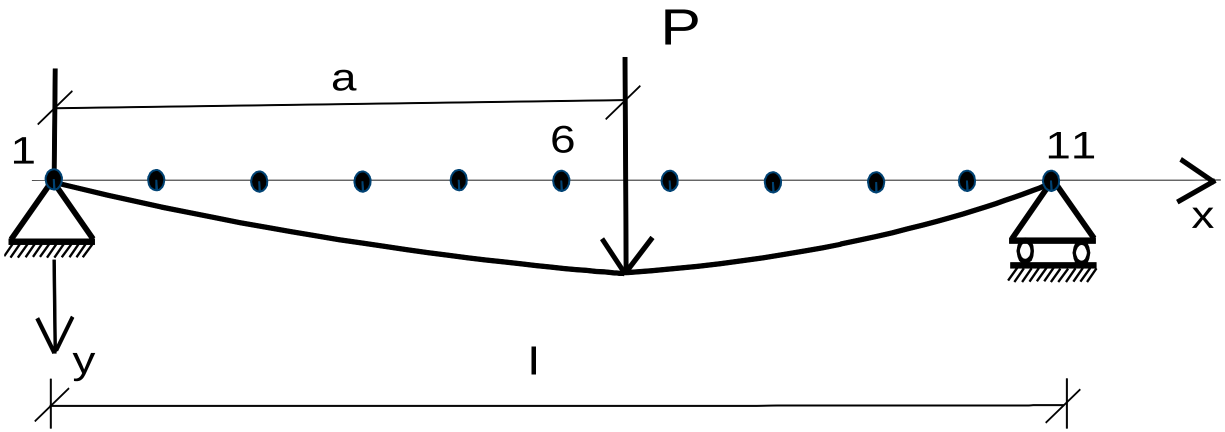
Solution is given by a linear system of equations in the form

$$\mathbf{M} \cdot \overline{\mathbf{A}^{*}} = \mathbf{R}$$

$$\mathbf{M} = \left(\overline{\mathbf{\Phi}}^{\mathbf{T}}, \overline{\mathbf{\Phi}}\right)$$
$$\mathbf{R} = \left(\overline{\mathbf{\Phi}}^{\mathbf{T}}, \mathbf{U}^{*}\right)$$

Finally, replacing missing elements of $\mathbf{U}^{*}$ by corresponding calculated elements of $\mathbf{U}^{**}$

Verification Example – GPOD



EI = 100000 kNm
l = 10 m
11 nodes
x = 0,1,2,...,10 m

P = 10,40,70,100 kN
a = 2,4,6,8 m

Analytical solutions :

$$y(x) = \frac{P(l-a)x}{6lEI}\left(l^2 - x^2 - (l-a)^2\right), \quad for\ 0 < x < a$$

$$y(x) = \frac{P(l-a)}{6lEI}\left(\frac{l}{l-a}(x-a)^3 + (l^2 - (l-a)^2)x - x^3\right), \quad for\ a < x < l$$

Verification Example – GPOD

16 snapshots (4 P x 4 a)

$$U = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0.5 & 1.9 & \dots & 2.2 & 3.2 \\ 0.9 & 3.4 & \dots & 4.3 & 1.1 \\ \dots & \dots & \dots & \dots & \dots \\ 0.3 & 1.3 & \dots & 3.3 & 4.7 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

11 nodes

16 snapshots

Truncated POD basis vectors $\bar{\Phi}$

$$\bar{\Phi} = \begin{bmatrix} 0 & 0 & 0 \\ 0.1 & -0.3 & 0.3 \\ 0.3 & -0.4 & 0.4 \\ \dots & \dots & \dots \\ 0.1 & 0.3 & 0.3 \\ 0 & 0 & 0 \end{bmatrix}$$

11 rows

3 columns

U*: solution of P = 125 kN and a = 2.5 m

$$U^* = \begin{bmatrix} ? \\ 6.68 \\ \dots \\ ? \\ 17.9 \\ ? \\ \dots \\ 4.83 \\ ? \end{bmatrix}$$

Node 2

Node 6

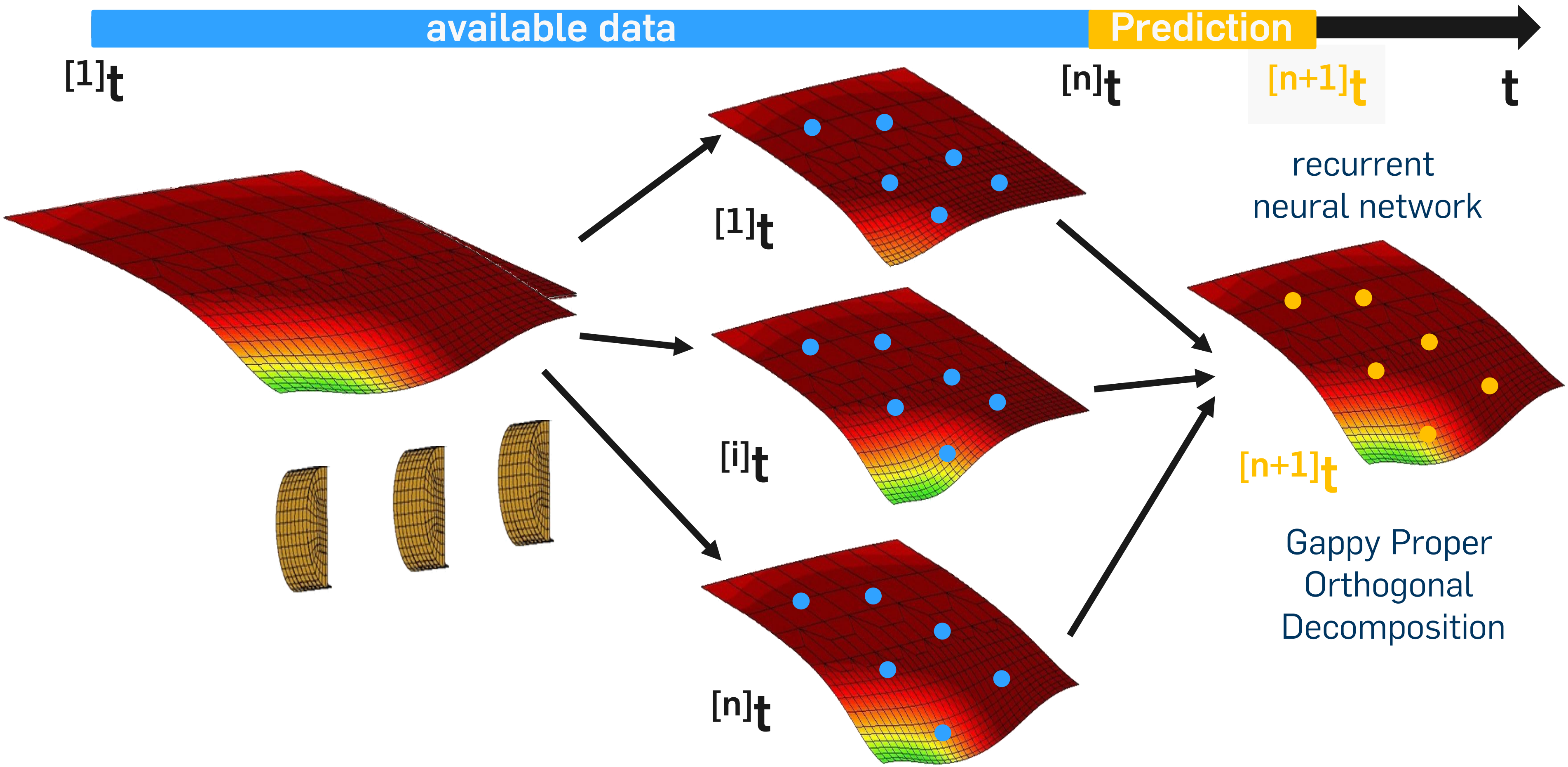
Node 10

GPOD

$$U^* = \begin{bmatrix} 0 \\ \mathbf{6.68} \\ \dots \\ 18.03 \\ \mathbf{17.9} \\ 16.17 \\ \dots \\ \mathbf{4.83} \\ 0 \end{bmatrix}$$

Error comparing with analytical solution is 0.34%

Hybrid RNN-GPOD surrogate model

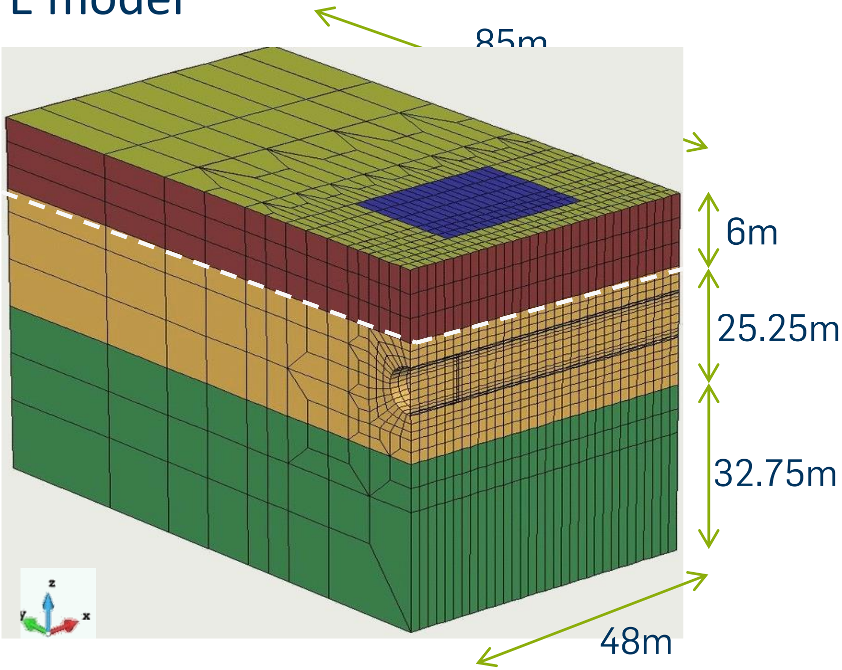


recurrent neural network

Gappy Proper Orthogonal Decomposition

Example – hybrid RNN-GPOD surrogate model

FE model



uncertain geotechnical parameter
Drucker-Prager Model

Parameter	E-Modul E_2 (MN/m^2)
Min	40
Max	130

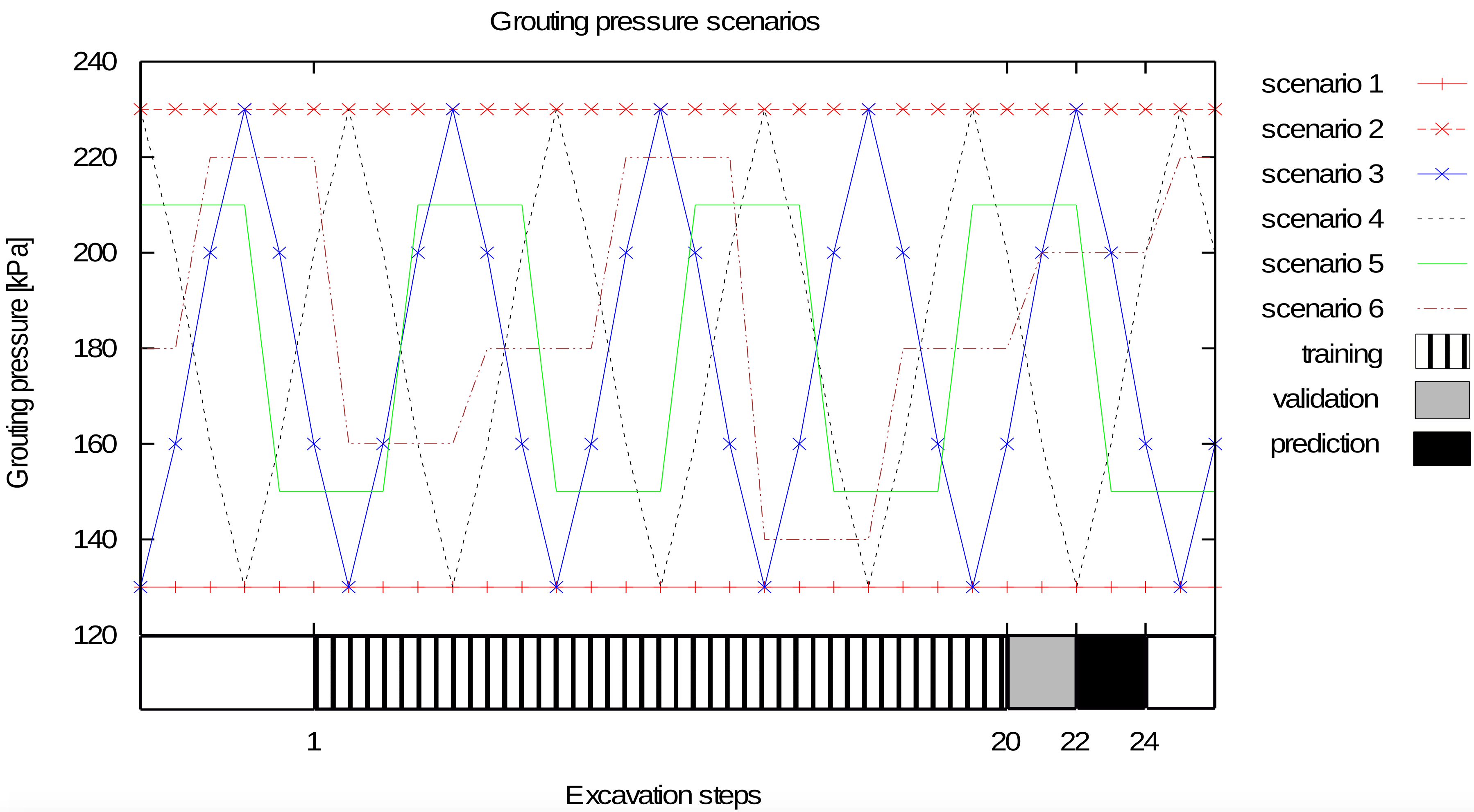
- hybrid process-oriented surrogate model – recurrent neural network in combination with Proper Orthogonal Decomposition Method (POD)

E_2
GP(t) ➞ $\underline{v}(t)$ time-varying settlement field

E_2: stochastic number

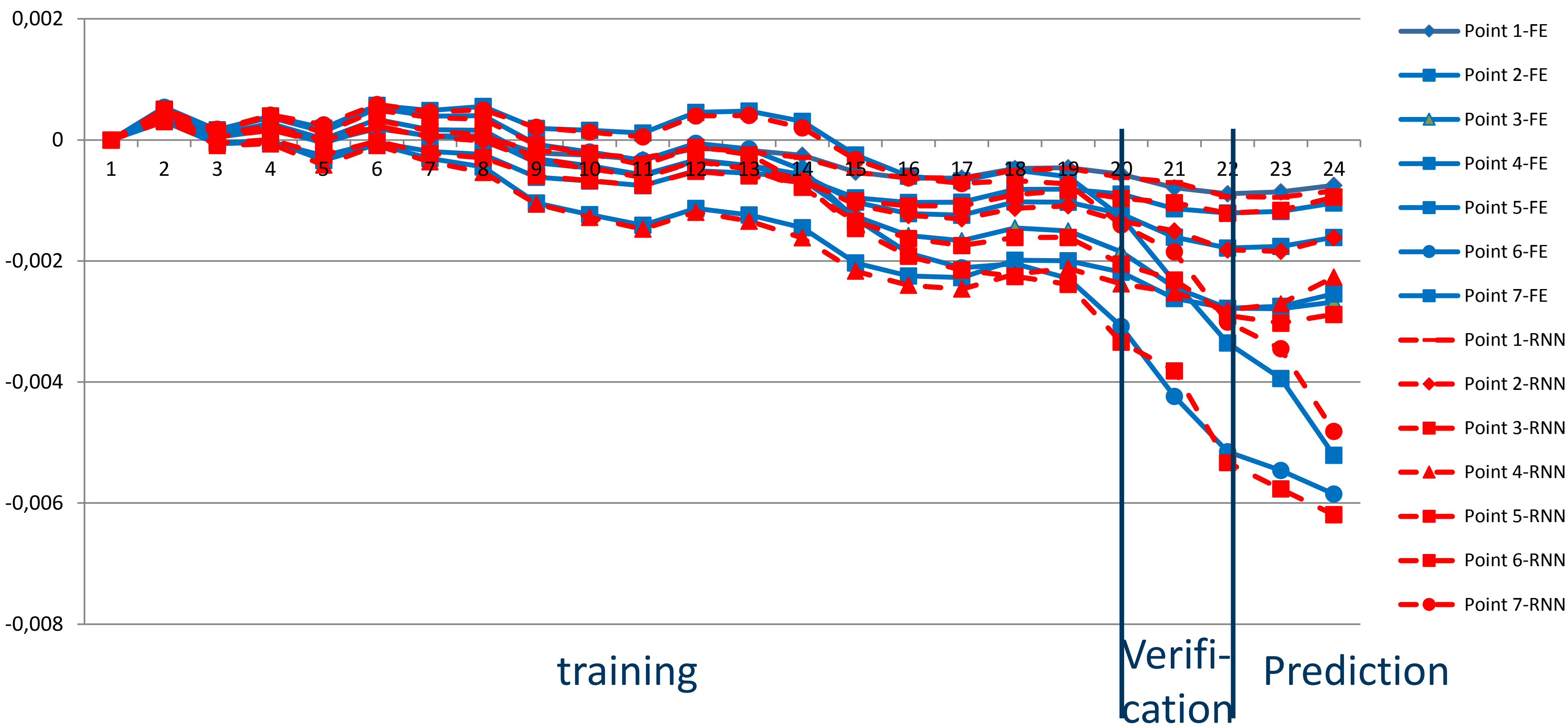
Example – hybrid RNN-GPOD surrogate model

Time varying grouting pressure



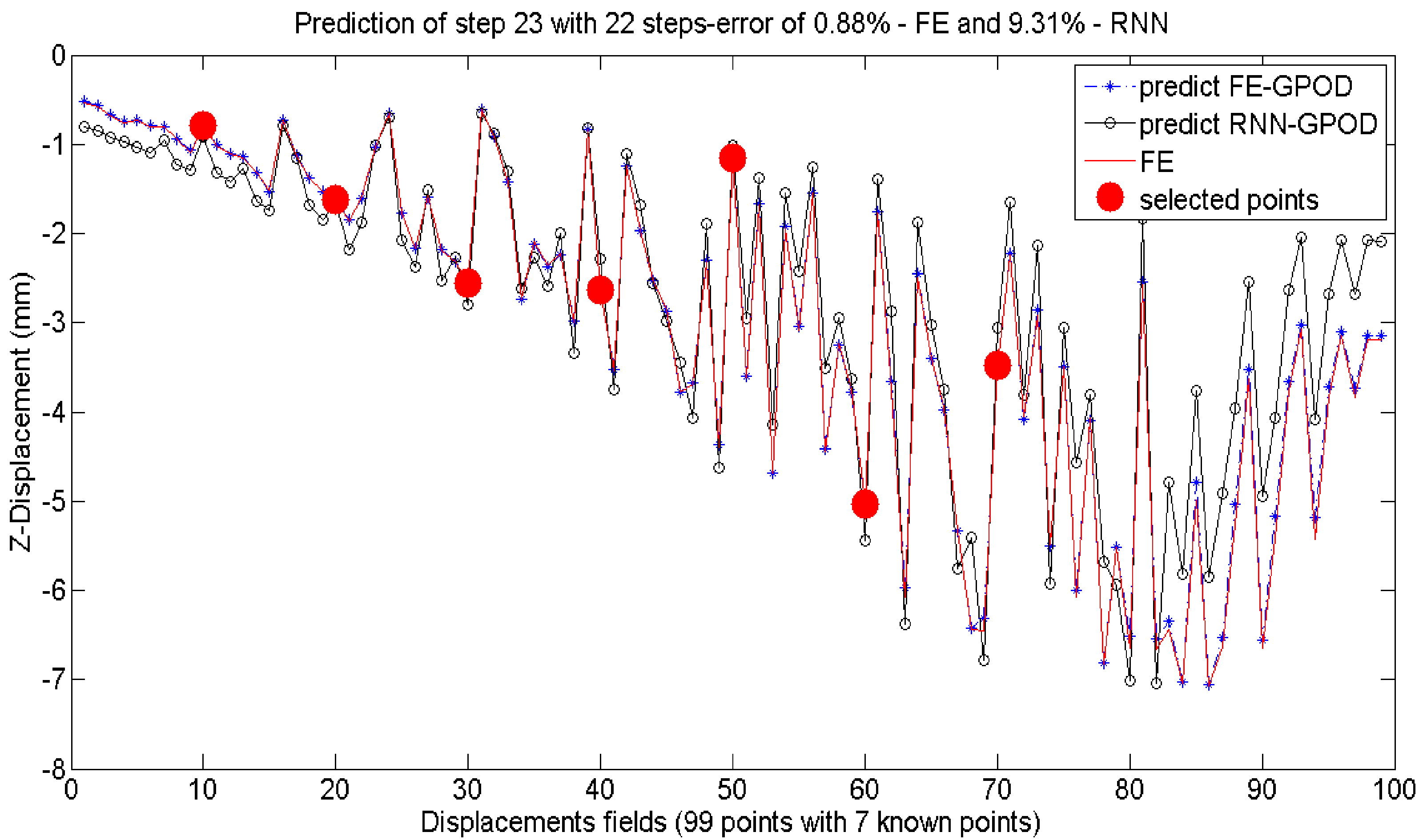
Example – hybrid RNN-GPOD surrogate model

Displacement processes of 7 monitoring points – Prediction with RNN



Example – hybrid RNN-GPOD surrogate model (stochastic analysis)

Displacement field predictions by GPOD-RNN (Case 36 : E_2 = 90 MPa, scenario 6)



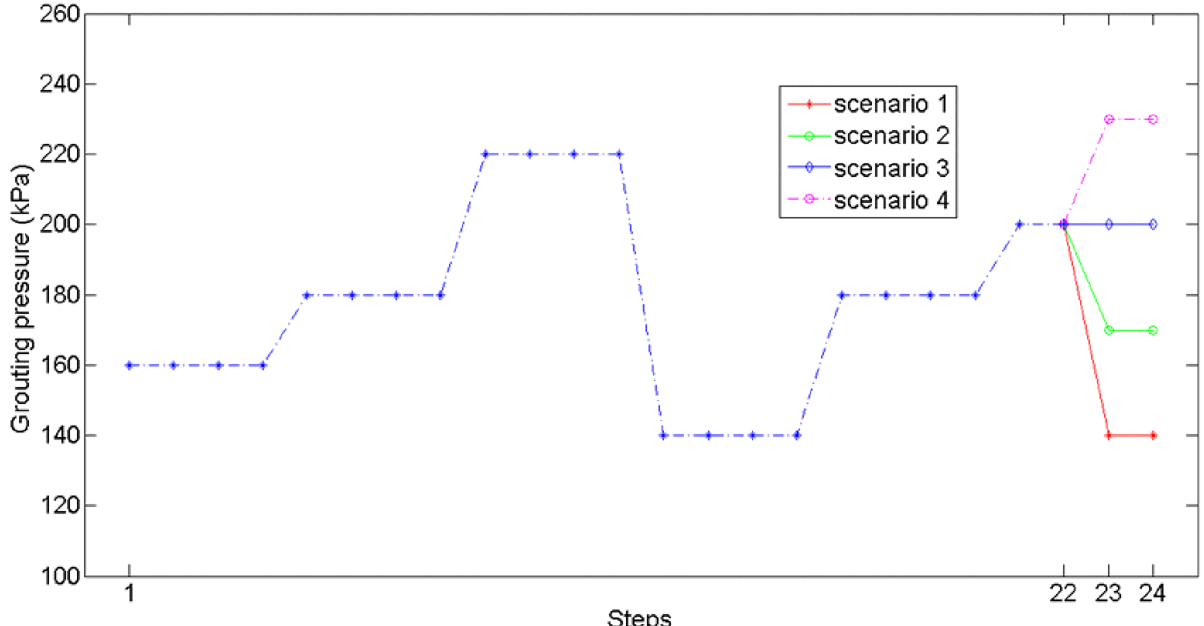
Example – hybrid RNN-GPOD surrogate model (stochastic analysis)

$$cdf: F(E_2) = \frac{1}{1+e^{-\left(\frac{E_2-a}{b}\right)}}$$

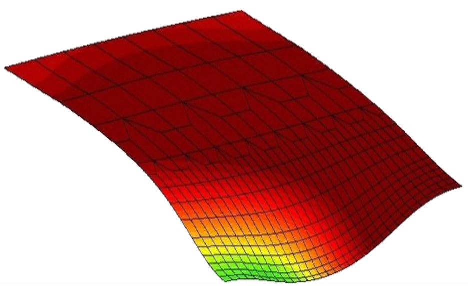
a = 80 MPa (mean value)
b = 3 Mpa

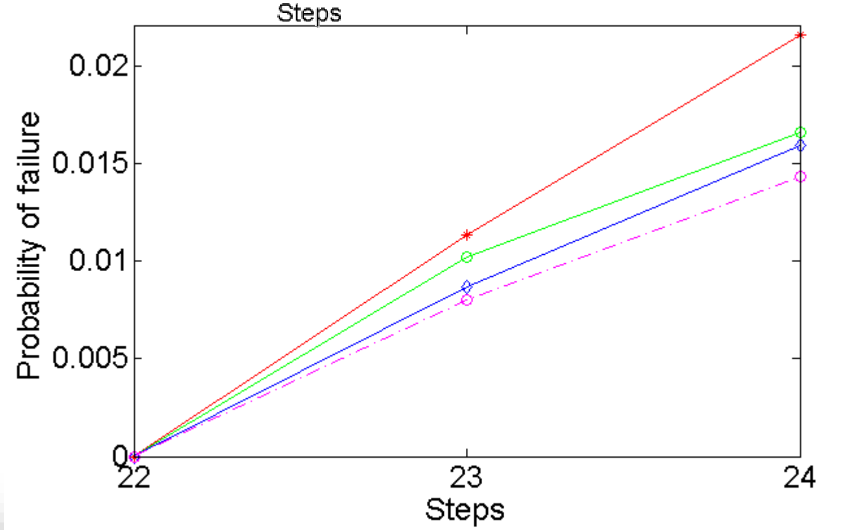
${}^{[n]}P$: scenario 6

limit state: 10 mm



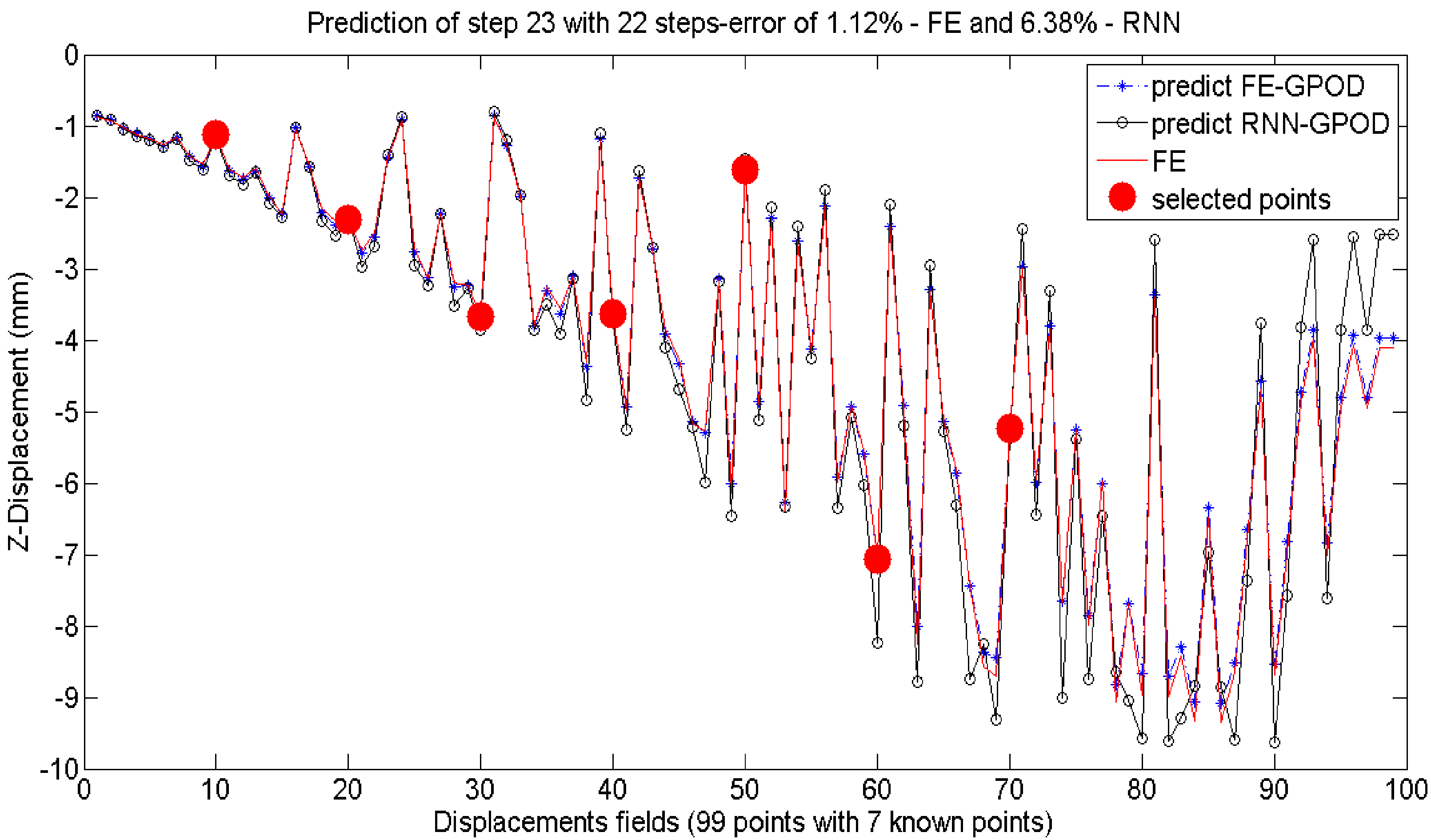
stochastic reliability analysis





Example – hybrid RNN-GPOD surrogate model (interval analysis)

Displacement field predictions by GPOD-RNN (Case 54 : E_2 = 70 MPa, scenario 5)



Example – hybrid RNN-GPOD surrogate model (interval analysis)

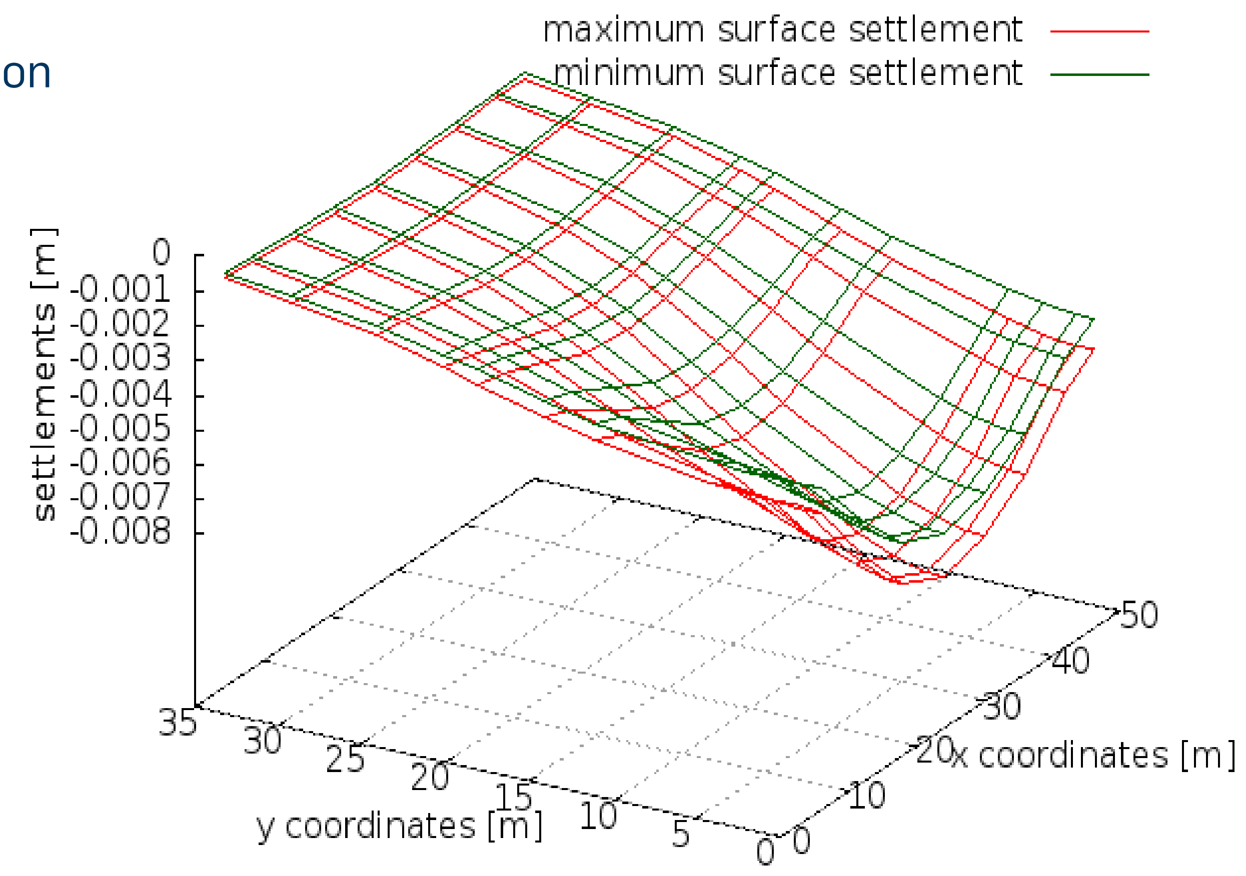
Interval $E_2 = [74, 86]$ MPa

Particle Swarm Optimization

Number of particles: 20

$c_1 = c_2 = 1.494$

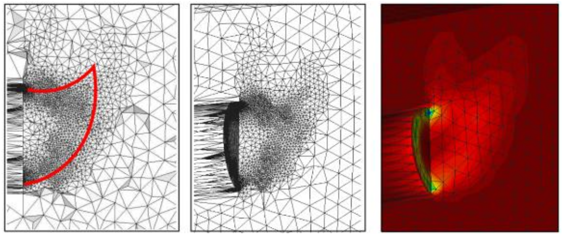
$c_3 = 0.729$



Summary / Outlook

- Process-oriented FE model for mechanized tunneling simulations
- Numerical simulation with uncertain geotechnical parameters
- Hybrid RNN-GPOD based surrogate model

- Further objectives:
  - damage risk assessment of existing buildings
  - stress reduction in tunnel lining
  - investigation of tunnel face stability



- Real-time predictions to support steering
- P-box approach for polymorphic uncertain data