Reliable Condition Assessment of Structures Using Uncertain or Limited Field Modal Data

Mojtaba Dirbaz
Mehdi Modares
Jamshid Mohammadi

Motivation

- ❑The primary goal of Structural Health Monitoring (SHM)
- ❑The application of SHM systems has become increasingly more popular.
- ❑However, most condition assessment methods have shortcomings particularly in conjunction with field measurements because:

(1) the available data is limited, and

(2) the inherent uncertainties in both experimental data and analytical methods lead to limited information about the condition of structures.

Golden Gate Bridge (San Francisco)

Outline

- Introduction
- Problem Definition
- Scopes and Objectives
- Methodology - Stochastic Structural Condition Assessment (SSCA)
- Numerical Example
- Summary and Conclusions

Introduction

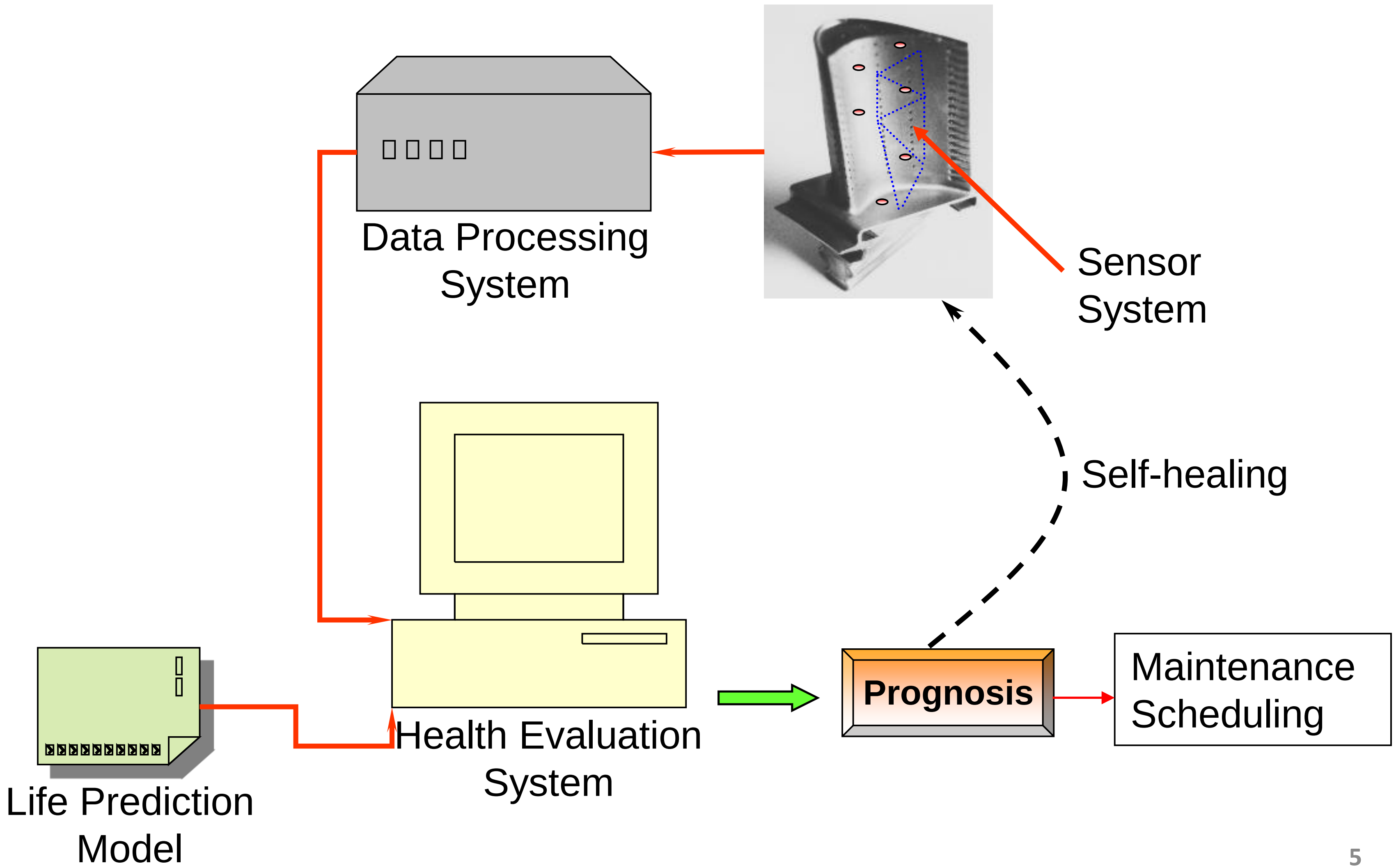
❑**<u>Purposes of SHM:</u>** Detect structure damages

- Safety, Safety, Safety
- Provide maintenance and rehabilitation advices
- Improve design guidelines
- Disaster mitigation

❑SHM usually compares a structure condition relative to a baseline state to **identify damage**

❑Determining the existence, location and extent of damage is critical to prevent any catastrophic failure as well as to enhance the structure's safety and serviceability.

Typical SHM System



Introduction

Monitoring of a structure in different levels

- **Level 1 (Raw data)**

  Displacement, Acceleration, Strain, Rotation, Corrosion, and other level 1 measurements

- **Level 2 (Identification stage)**

  Natural frequency, Statistical patterns , and other level 2 measurements

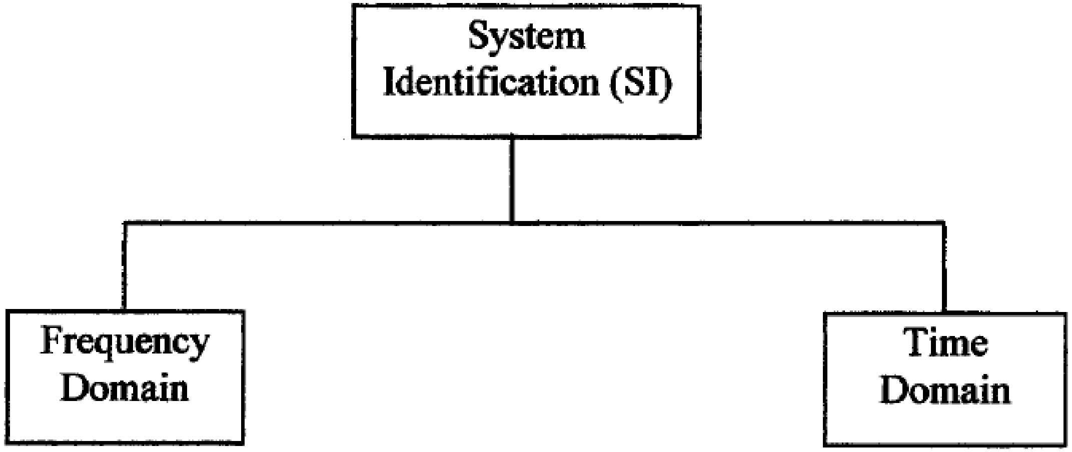
- **Level 3 (Considerable change)**

  Flexibility, Curvature, Reliability index and other level 3 measurements

- **Level 4 (Pridiction)**

  Bayesian updating, and other level 4 measurements

Introduction



Classification of the system identification

- The modal properties (natural frequencies and mode shapes) are function of dynamic properties (mass and stiffness) of the structure.
- The presence of defects is expected to change the dynamic properties and thus the defective structures can be identified using this approach.

Problem Definition

Concept of system identification (from Toki et. Al 1989)

❑A typical System Identification(SI) technique has three components; input excitation, the system, and the output response information.

➢**The problem is**

❖Most condition assessment methods have shortcomings particularly in conjunction with field measurements and may produce limited results.

❖These issues can be attributed to 1) measurement noise 2) incompleteness of modal data, and 3) uncertainties inherent in experimental data and analytical methods.

Scopes and Objectives

- To determine a new vibration-based probabilistic method for condition assessment of structures through a Monte-Carlo simulation procedure.

- To determine the bounds on extent and location of damage for structures.

Methodology - Stochastic Structural Condition Assessment (SSCA)

The general algorithm for the method of Stochastic Structural Condition Assessment (SSCA) consists of the following steps:

- Step 1: Construct initial FE model
- Step 2: Determine the uncertainty in the modal data
- Step 3: Perform Monte-Carlo simulations for stochastic condition assessment of the structure

Step 1 of SSCA : Construct initial FE model

- Model the intact structure based on design drawings and available information and/or field measurements

- Determine the intact structure's stiffness and mass matrices.

- Determine the intact structure's modal characteristics (e.g. natural circular frequencies and mode shapes)

$$K_u \Phi_{ui} = \omega_{ui}^2 M_u \Phi_{ui} \qquad i = 1, \ldots, n$$

Step 2 of SSCA: Determine the uncertainty in the modal data

- Collect a series of sensor-based modal measurements (e.g. natural frequencies using accelerometers).

- Quantify the uncertainties inherent in the measured data as bounded random variables using statistical methods.

$$\tilde{\omega}_{di} = [\mu - \delta, \mu + \delta] \qquad i = 1, \dots, n$$

Step 3 of SSCA: Perform Monte-Carlo simulations for stochastic condition assessment of the structure

- Determine the FE model of damaged structure by obtaining structure's stiffness matrix using modal data and intact model through an iterative optimization scheme.

- Determine the extent and location of damage based on changes between the structural element stiffness for the "as built condition" and "damaged condition".

- Determine the bounds on both location and extent of damage that caused the degradation of the system (using the Monte-Carlo simulations results).

Step 3 of SSCA: Perform Monte-Carlo simulations for stochastic condition assessment of the structure (Cont.)

3.1 FE Modeling for Damaged Structure for Each Realization

In each realization, the generalized eigenvalue problem for the damaged structure is:

$$K_d \Phi_{di} = \omega_{di}^2 M_d \Phi_{di} \qquad for\ i = 1, \ldots, m$$

Since local damage does not have significant effects on changing the structure’s mass, it is assumed that

$$M_u = M_d$$

$$K_d = K_u - \sum_{j=1}^{e} \alpha_j K_j$$

Step 3 of SSCA: Perform Monte-Carlo simulations for stochastic condition assessment of the structure (Cont.)

3.1 FE Modeling for Damaged Structure for Each Realization

$$(\Phi_{ui})^T K_d \Phi_{di} = \frac{\omega_{di}^2}{\omega_{ui}^2}(\Phi_{ui})^T K_u \Phi_{di}$$

Combination of all equations yields:

$$\sum_{j=1}^{n} \alpha_j (\Phi_{ui})^T K_j \Phi_{di} = (1 - \frac{\omega_{di}^2}{\omega_{ui}^2})(\Phi_{ui})^T K_u \Phi_{di}$$

$$\sum_{j=1}^{e} \alpha_j C_{j,i} = b_i$$

Step 3 of SSCA: Perform Monte-Carlo simulations for stochastic condition assessment of the structure (Cont.)

3.2 Optimization Scheme

- In order to determine all α_j , a least square approach (optimization scheme) is used to compute all α_j which must satisfy the constraints.

- The problem is a linear constrained optimization solved by the MATLAB optimization toolbox using the LSQNONNEG routine.

$$\min_x \|Cx - b\|_2^2 \text{ , } where\ x \geq 0$$

Step 3 of SSCA: Perform Monte-Carlo simulations for stochastic condition assessment of the structure (Cont.)

3.3 Identification of Location and Extent of Damage

- In each realization, all values of α_j, which represent the extent of damage for a specific element, are computed.

- The index of this variable (j) represents the location of damage and the value of α_j represents and extent of damaged.

- For example, $\alpha_3 = 0.35$ means that extent of damage for element 3 is 35% .

3.4 Using the Monte-Carlo simulation results, the bounds on both location and extent of damage are determined.

Numerical Example (SHM benchmark problem)

- SSCA is applied to a structural health monitoring benchmark model problem given jointly by the International Association for Structural Control and the dynamics committee of the American Society of Civil Engineers (IASC-ASCE).

- The benchmark problem is a four-story two-bay by two-bay steel braced frame. It has a 2.5m × 2.5m plan, and is 3.6m tall. The structural elements are hot-rolled grade 300 W steel with a nominal yield stress 300 MPa (42.6 ksi).

Numerical Example (SHM benchmark problem) (Cont.)

Table I. Properties of structural members

Property	*Columns*	*Floor beams*	*Braces*
Section type	B100x9	S75x11 0.15	L25x25x3
Cross-sectional area A (m^2)	$1.133\text{x } 10^{-3}$	$1.43\text{x } 10^{-3}$	$0.141\text{x } 10^{-3}$
Moment of inertia (strong direction) I_y (m^4)	$1.97\text{x}10^{-6}$	$1.22\text{x}10^{-6}$	0
Moment of inertia (weak direction) I_z (m^4)	$0.664\text{x}10^{-6}$	$0.249\text{x}10^{-6}$	0
St. Venant torsion constant J (m^4)	$8.01\text{x}10^{-9}$	$38.2\text{x}10^{-9}$	0
Young's modulus E (Pa)	$2\text{x}10^{11}$	$2\text{x}10^{11}$	$2\text{x}10^{11}$
Shear modulus G (Pa)	*E*/2.6	*E*/2.6	*E*/2.6
Mass per unit volume ρ (kg/m^3)	7,800	7,800	7,800

Numerical Example (SHM benchmark problem) (Cont.)

Table II. Horizontal story stiffness (MN/m) of undamaged model.

Story	DOF	Undamaged Stiffness
1	x	106.60
1	y	67.90
1	θ	232.02
2	x	106.60
2	y	67.90
2	θ	232.02
3	x	106.60
3	y	67.90
3	θ	232.02
4	x	106.60
4	y	67.90
4	θ	232.02

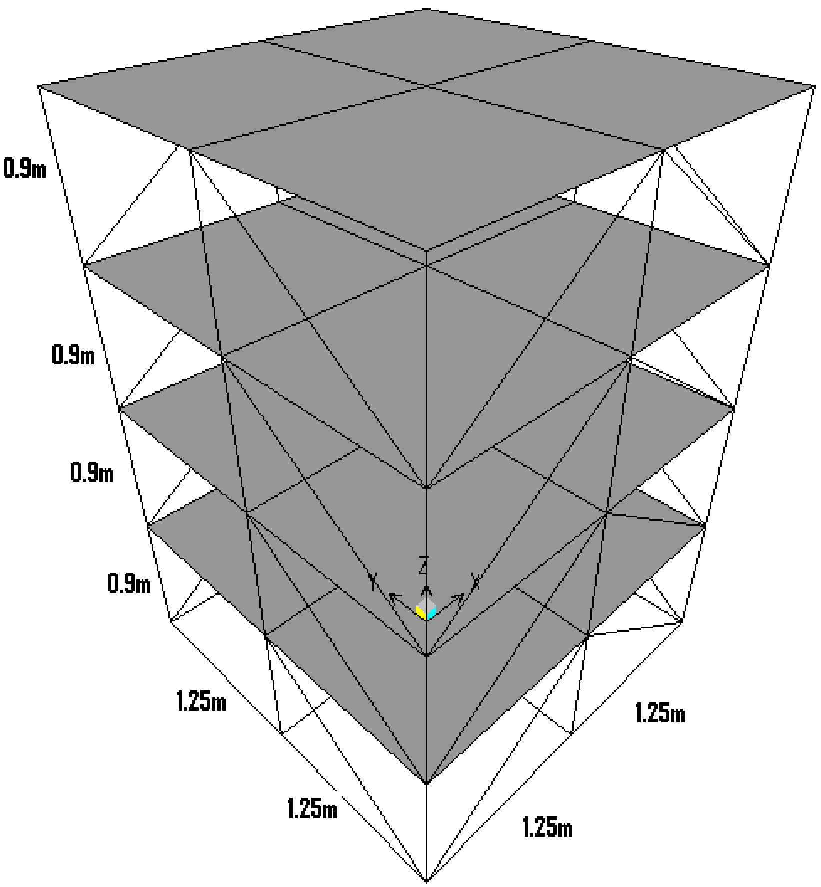


Diagram of the analytical model

Numerical Example (SHM benchmark problem) (Cont.)

Damage Patterns. In this study, two damage patterns (two cases) are investigated.

Case 1: No stiffness in the braces of the first story (i.e., removing all braces but considering the contribution of mass), and

Case 2: No stiffness in all braces of the first and third stories. (i.e., all braces in first and third stories are removed).

Numerical Example (SHM benchmark problem) (Cont.)

Table III. Mean values and bounds on natural circular frequencies of the damaged structure.

Mode	Mean (rad/s)	Lower and Upper Bounds (rad/s)
1	36.63	[34.80 , 38.46]
2	59.80	[56.81 , 62.79]
3	69.93	[66.43 , 73.43]
4	93.80	[89.11 , 98.49]
5	156.93	[149.08 , 164.77]
6	180.84	[171.80 , 189.89]
7	227.95	[216.55 , 239.35]
8	261.66	[248.58 , 274.74]
9	295.71	[280.93 , 310.50]
10	344.06	[326.86 , 361.27]
11	407.50	[387.13 , 427.88]
12	466.67	[443.34 , 490.00]

Numerical Example (SHM benchmark problem) (Cont.)

Table IV. Exact values and bounds of percent loss in horizontal story stiffness.

Story	DOF	Exact	Lower and Upper Bounds
1	x	45.24%	[23.97% , 50.49%]
1	y	71.03%	[70.99% , 93.41%]
1	θ	64.96%	[4.59% , 79.16%]
2	x	0	[0 , 9.57%]
2	y	0	[0 , 0]
2	θ	0	[0 , 0]
3	x	45.24%	[26.82% , 50.57%]
3	y	71.03%	[49.32% , 72.56%]
3	θ	64.96%	[39.25% , 65.33%]
4	x	0	[0 , 9.67%]
4	y	0	[0 , 11.95%]
4	θ	0	[0 , 32.27%]

Summary and Conclusions of SSCA

- A new method for damage detection and assessment of structures using uncertain modal data, referred as Stochastic Structural Condition Assessment (SSCA), is developed.
- SSCA applies finite element analyses and an optimization scheme along with Monte-Carlo simulation for more accurately assessing the condition of a structure with uncertain or limited data.
- The main conclusions of the study are:

1. In the presence of uncertainty in the measured modal data, SSCA is capable of determining the bounds on both location and extent of any possible damage based on the change of structural stiffness occurred from the "as built condition" to the "damaged condition."

2. Numerical illustration shows that SSCA, because of consideration of uncertainties, is capable of identifying the bounds on location and extent of damage more precisely.

Thank you

Questions and comments?