Random Number simulation for the Correlation between the Allowable stress design and the Reliability Analysis

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Abstract: This paper visualizes the probability distributions which are in the background of the allowable stress design code of the bridges. Random vehicle weight samples are generated as for several Traffic Flow Models, and they comprise random arrayed motorcades. Various traffic loads for a lane are calculated, and their variances are obtained. Thus the girder stress dispersion is identified as the action side distribution. Reliability Assessment of the Bridge Girder can be carried out through the convolution of this action stress distribution and the yield stress distribution. Accordingly, the allowable stress design method can be converted to the Reliability analysis design. This Reliability Assessment for the Bridge Girder which is designed according to the allowable stress method derives Reliability Index about 6.0. The fragility of this assessment does not include the uncertainties such an aged damage or the error. Then the deterioration probability is assumed and the random samples of defects are generated. The Reliability Probability is calculated using those samples. The correlation between the allowable stress design method and the Reliability Analysis is clarified.

Keywords: Performance-based Design, Probability density distribution, Reliability Index

1. Introduction

Probability distribution of action is necessary for the reliability analysis design. Before all, the live-load probability distribution is the most important issue for designing a bridge structure based on the reliability theory. Some sort of assumption must be introduced in a present situation.

In allowable stress design method, nominal load values and allowable stress values should be decided based on some actual data. Even if it was done upon experimental judgment and has no specific derivation process, that includes probabilistic consideration. In Reliability Analysis, the load action and the material strength are treated as probability density distributions. This paper discusses the conversion way to the Reliability Analysis.

2. Concept of Conversion to Reliability Analysis

In allowable stress design method, the acting stress caused by the nominal load must be restricted within the allowable range. The allowable stress is decided by dividing the yield stress or the ultimate strength by the given safety factor, depending on the steel classification.

In order to convert this way into Reliability Analysis, probability distribution of the acting stress and the yield stress are characterized as in Figure-1, on the stress domain. This figure shows an example for steel girder stress which is built with SM400(JIS). The position relationship between the nominal allowable stress and the variability of yield stress, and the relation between acting stress by nominal loads and its expected variability, are shown.

The allowable stress for the usual steel material is decided considering the safety factor of about 1.7 against yielding stress. This is a substance commentary concept of Japanese "Specification of Highway Bridges1)"(SHB). Figure-1 is characterized based on this concept.
At this point, the variability is described as a normal distribution. Simplification is emphasized.

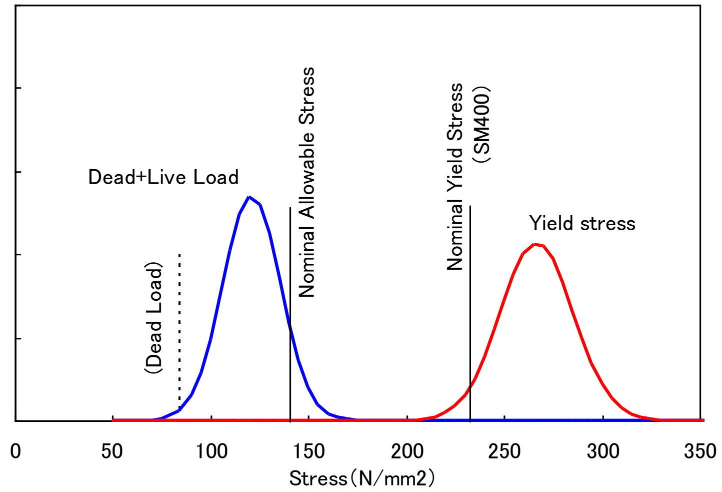

Figure-1 Concept of probability distributions

3. Probability Density Distribution of Traffic Load

3.1 Random Number Generation for Traffic Load

This paper makes a proposal method for setting the live-load probability distribution using random vehicle weights. Traffic Flow Models are adopted which are propounded in Japanese "Fatigue design Guide Line for Road Bridges2)" as the reference for examining the fatigue of the bridge.
In the Traffic Flow Models, vehicles are classified into eight categories which have hierarchal component percentages. Each category has the separate weight probability distribution. The component percentages of classified vehicles are shown in Table-1, in accordance with Traffic Flow Models which are classified into five classes A to E. And each vehicle category has their weight probability distribution shown in Table-2.

Table-1 Component of Traffic Flow Classes

Traffic Flow	A	B	C	D	E
Passenger car	0.776	0.653	0.551	0.490	0.327
Small truck	0.174	0.147	0.124	0.110	0.073
Medium truck	0.020	0.081	0.131	0.161	0.242
Large truck	0.016	0.068	0.103	0.127	0.190
Large dump truck	0.005	0.018	0.030	0.037	0.056
Tank truck	0.003	0.011	0.017	0.021	0.032
Semi-trailer	0.005	0.021	0.034	0.042	0.063
Large Bus	0.001	0.006	0.010	0.012	0.017

Table-2 Probability of Vehicle Weight (Ton)

Categories	μ	σ	Max.	Min.
Passenger car	1.30	0.36	3.10	0.20
Small truck	3.62	1.31	12.60	1.00
Medium truck	6.19	2.50	18.30	1.60
Large truck	16.75	6.29	45.20	3.80
Large dump truck	19.62	9.54	61.90	5.40
Tank truck	13.82	6.31	33.70	3.90
Semi-trailer	24.78	12.92	86.70	5.10
Large Bus	13.84	2.41	18.70	6.80

(All Log-Normal Probability Distribution)

In order to simulate the sequential vehicles on the bridge, Random Numbers are generated for each category in accordance with the probability distributions and the component percentage. (It is to be noted that these distributions have the maximum and minimum limit, so they are not Log-Normal distribution in their tail region.)

Generated samples are merged and shuffled. Thus a random arrayed motorcade is built for each Traffic Flow Model. An assumption is herein introduced that the generated motorcade samples have the similar probability distribution to that of traffic loads in the whole bridge life.

Figure-2 shows the histogram of all samples in case of Traffic Flow class-E, as the example of the heaviest traffic.

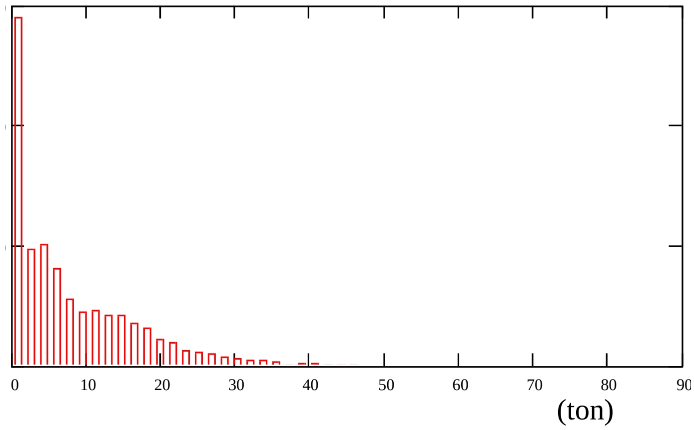

Figure-2 Histogram of All samples

The nominal load code of the Japanese "Specification of Highway Bridges" is regulated as the uniform distributed load and the concentrated load. Above samples are converted into the uniform distributed loads, and are compared to the nominal loads.

Distributed load is calculated as follows;

Serial vehicle weight of the random arrayed motorcade is counted until its summed length reaches the loading length. Repeating this from the head to the next sequentially, the summed weights are divided by the loading length. Thus the distributed load intensities are derived. Therefore its probability distribution characteristics are obtained, and listed in Table-3 in accordance with loading length.

Table-3 Characteristics of Distributed Load (t/m/lane)

Traffic Flow		L=30	L=40	L=50	L=60
A	Mean	0.416	0.417	0.416	0.417
	Median	0.342	0.360	0.385	0.403
	σ	0.234	0.185	0.132	0.093
	V	0.563	0.444	0.317	0.223
B	Mean	0.654	0.655	0.657	0.658
	Median	0.565	0.598	0.628	0.644
	σ	0.363	0.297	0.219	0.158
	V	0.555	0.453	0.333	0.240
C	Mean	0.841	0.846	0.833	0.847
	Median	0.769	0.801	0.818	0.834
	σ	0.428	0.343	0.234	0.177
	V	0.509	0.405	0.281	0.209
D	Mean	0.932	0.939	0.940	0.940
	Median	0.863	0.895	0.920	0.928
	σ	0.435	0.350	0.251	0.178
	V	0.467	0.373	0.267	0.189
E	Mean	1.148	1.151	1.159	1.154
	Median	1.092	1.113	1.143	1.145
	σ	0.436	0.347	0.258	0.179
	V	0.380	0.301	0.223	0.155

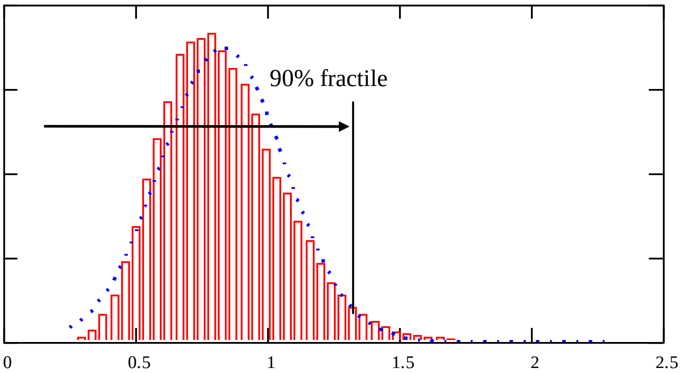

Figure-3 Histogram of distributed load （tonf/m/lane）

Figure-3 shows the histograms of distributed load intensities of Traffic flow C. The dotted line is the Normal distribution fit line which has the mean value and the standard deviation of Traffic flow C.
The mean values of the distributed load shown in Table-3 are larger than the median values. And the shape of the histogram is almost similar to the Log-Normal distribution.
Probability distribution characteristics of the merged samples are not strictly the Log-Normal distribution because the tails of the individual distributions were cut off. In this paper, it was ruled that the approximation by Normal-Distribution should be the best solution for the purpose of simplification.

Several fractile values of the distributed loads are calculated and compared with the nominal load value of the Japanese design code "Specification of Highway Bridges" (SHB). Through those simulations, regulations of uniform load reduction for the span length and that of transverse direction can be explained theoretically.

3.2 Variation by Loading Length

As shown in Table-3, mean values of distributed load intensity are mainly correspondent to traffic flow class and not so effected by loading length. The coefficients of variance($V=\sigma/\mu$) reduce with loading length. The longer the loading length, the more various weights of vehicles are included in the loading length, then the coefficient of variance of distributed load decreases. That obeys the Law of Large Numbers. If the loading length becomes infinity, variance becomes zero.
Using these characteristics, the expectations on several fractile percentages are calculated. They are shown in Figure-4, together with the nominal uniform load value of the Japanese design code. This Figure-4 shows the relative position between simulated expectation and the nominal load value.
Higher the fractile percentage, the reduction slope of expectation becomes larger. Among these results, we can know that the nominal load value corresponds to which percentages expectation value. For example, traffic flow C line of 90% fractile is the nearest to the nominal load line. And it may not sound strange to our empirical recognition that nominal load may be 90% fractile or so.

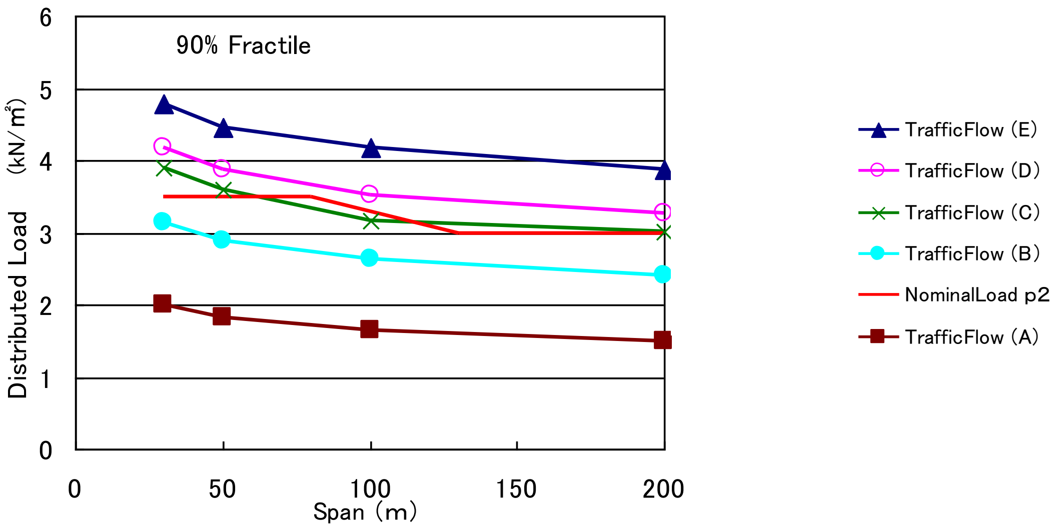

Figure-4 Distributed load on loading length

3.3 Variation by Number of Loading Lanes

The reduction effect of the fractile according to the loading width exists on the same reason as loading length. Figure-5 shows the transverse reduction regulation ruled by Japanese design code "SHB".

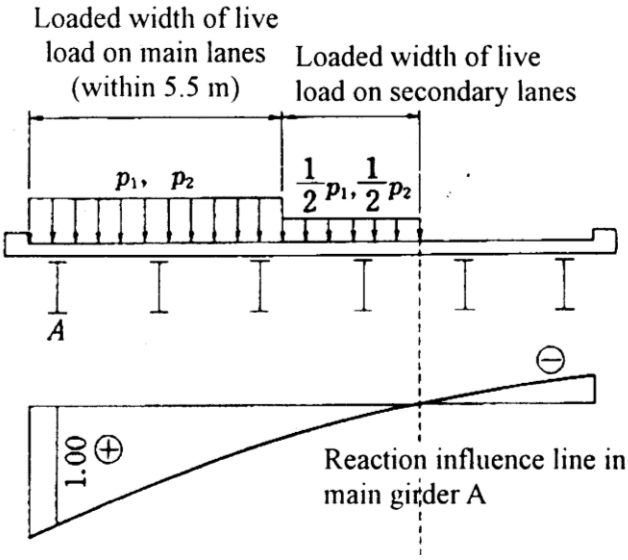

Figure-5 Transverse Reduction of L-Load (SHB)

The load reduction regulation according to loading width is based on the same reason as to loading length. Then, the problem can be analyzed such as the loading length is multiplied by the lane number. The same simulations as previous section for the loading length of from 30m to 180m were carried out. The results are shown in Table-4.

Table-4 Characteristics of Distributed Load by increase of number of lanes (ton/m/lane)

L=		30	60	90	120	150	180
A	Mean	0.416	0.417	0.416	0.416	0.416	0.417
	Median	0.342	0.367	0.381	0.389	0.395	0.401
	σ	0.234	0.169	0.138	0.121	0.108	0.093
	V	0.563	0.405	0.332	0.291	0.260	0.235
B	Mean	0.654	0.667	0.668	0.668	0.668	0.668
	Median	0.565	0.622	0.631	0.640	0.651	0.651
	σ	0.363	0.282	0.232	0.201	0.181	0.167
	V	0.555	0.423	0.347	0.301	0.271	0.250
C	Mean	0.841	0.847	0.848	0.847	0.847	0.847
	Median	0.769	0.809	0.819	0.825	0.825	0.831
	σ	0.428	0.314	0.259	0.226	0.203	0.186
	V	0.509	0.371	0.305	0.267	0.240	0.220
D	Mean	0.932	0.939	0.939	0.939	0.939	0.940
	Median	0.863	0.901	0.917	0.924	0.929	0.926
	σ	0.435	0.321	0.265	0.230	0.205	0.188
	V	0.467	0.342	0.282	0.245	0.218	0.200
E	Mean	1.148	1.156	1.158	1.158	1.158	1.159
	Median	1.092	1.126	1.134	1.143	1.148	1.148
	σ	0.436	0.324	0.270	0.235	0.211	0.193
	V	0.380	0.280	0.233	0.203	0182	0167

Focusing on the coefficients of variance in Table-4, those decrease in accordance with loading length almost independently of Traffic Flow class. And the reduction tendency of coefficient of variance can be approximated as it is inverse proportion to square of loading length. (Table-5)

Table-5 Reduction Rate of coefficients of variance by loading length

L=	30	60	90	120	150	180
Traffic Flow A	1.0	0.72	0.59	0.52	0.46	0.42
Traffic Flow B	1.0	0.76	0.62	0.54	0.48	0.45
Traffic Flow C	1.0	0.72	0.60	0.52	0.47	0.43
Traffic Flow D	1.0	0.73	0.60	0.52	0.46	0.42
Traffic Flow E	1.0	0.73	0.61	0.52	0.47	0.43
Mean	1.0	0.73	0.61	0.53	0.47	0.43
$\sqrt{30/\boldsymbol{L}}$ Applox.	1.0	0.71	0.58	0.50	0.44	0.41

Nevertheless those results may be seen which are inductively derived, they obey the Law of Large Numbers; "Variance of the sample which are picked out from the infinite population is inverse proportion to its number"

In accordance to this law, the increase of number of the lane can be treated same as the loading length is multiplied. The fractile value is derived as follows;

$$\boldsymbol{S} = \mu \left(1 + \boldsymbol{k} \cdot \boldsymbol{V} / \sqrt{\boldsymbol{n}}\right) \quad \cdots \quad (1)$$

$$\begin{cases} \boldsymbol{S} & : \text{Fractile value} \\ \mu & : \text{mean} \\ \boldsymbol{k} & : \text{Coefficient of shift} \\ \boldsymbol{V} & : \text{Coefficient of variance} \\ \boldsymbol{n} & : \text{Number of lanes} \end{cases}$$

It means the coefficient of variance decreases, accordingly the fractile value decreases by it. And, the mean uniform value of all lanes is described as follows:

$$\boldsymbol{p_m} = \boldsymbol{p}_1 \cdot \frac{1 + \dfrac{\boldsymbol{V_p}}{\sqrt{\boldsymbol{n}}} \cdot \boldsymbol{k}}{1 + \boldsymbol{V_p} \cdot \boldsymbol{k}} \quad \cdots \quad (2)$$

$$\begin{cases} \boldsymbol{V_p} & : \text{Coefficient of variance when single lane} \\ \boldsymbol{p}_1 & : \text{Fractile load value when single lane} \end{cases}$$

Consequently, the fractile load value of the first lane is defined same as when single lane, and that of second or later lane is derived as follows;

$$\boldsymbol{p_n} = \boldsymbol{n} \cdot \boldsymbol{p_a} - (\boldsymbol{p}_1 + \cdots + \boldsymbol{p_{n-1}}) \quad \cdots \quad (3)$$

$\boldsymbol{p_n}$: Fractile load value of nth lane

Figure-6 shows the reduction coefficients of the fractile value by number of lanes, when 90% fractile of traffic flow C is selected.

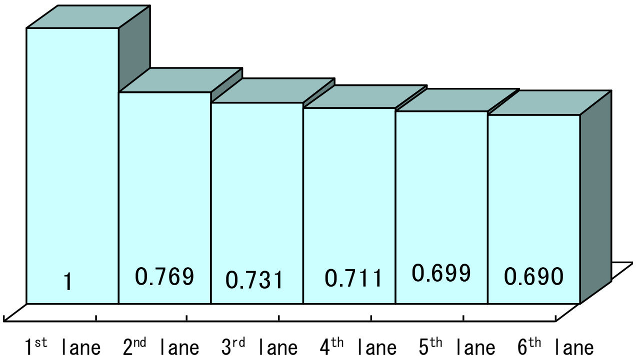

Figure-6 Reduction of fractile value by Number of lanes

4. Example Model Bridge

4.1 Dimension of the Model Bridge

In attempt to realize the tangible analysis example, several example model bridges are taken. The object bridge is described in Figure-7. At first, the allowable stress design method is applied in this section. Main structure of the bridge is as follow;

Bridge Type	: Simple Steel Girder with RC deck
Road Width	: 10.5 (m)
Span Length	: 30.0 ~ 60.0 (m)
Number of Girders	: 5
Steel material	: SM400 (JIS), f_{ta} ; 140 (N/mm2)

The change of the steel girder section is focused to simplifying the problem. The girder section is decided by the dominant load case "Dead Load +Live Load".

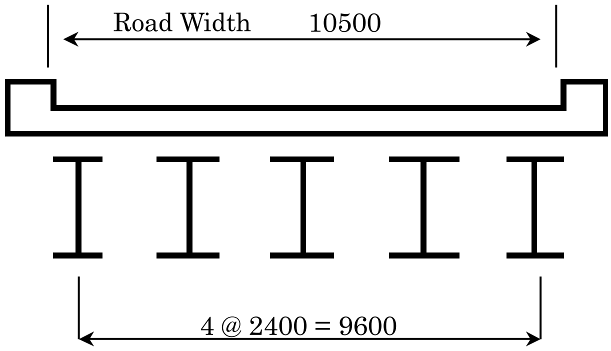

Figure-7 Cross section of the object Bridge

4.2 Design by Allowable Stress Method

The dimension of the girder section is decided by the bending moment. The nominal Load condition is applied in the allowable stress design. According to the regulation of the Japanese design code, Live Load intensities are set corresponding to the span length. Dead load is set based on the existing similar cases, as shown in Table-6.

Table-6 Nominal Load condition (kN/m^2)

		L=30	L=60
Dead Load	Deck	7.7	
	Girder	1.8	3.6
Live Load (uniform)	***p***1	10.0	
	p2	3.5	

Two kinds of uniform load are applied with the Loading Pattern for the girder such as shown in Figure-8. load is overlapped at the maximum point of the influence line.

The design results are shown in Table-7.

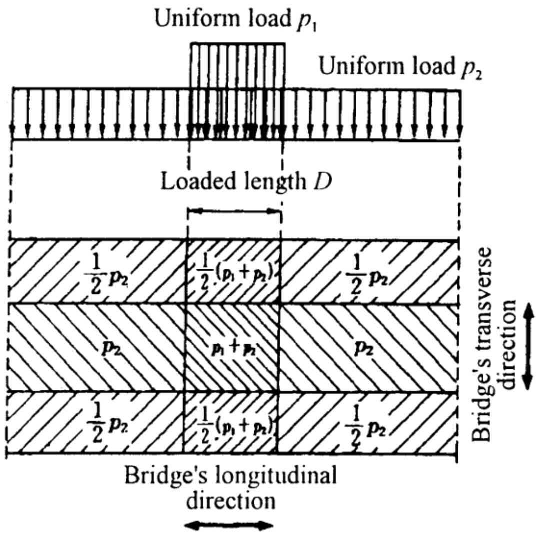

Figure-8 Loading Pattern of L load (SHB)

Table-7 Results of Allowable Stress Design

Span (m)	30	40	50	60
Bending Moment (kN*m)	23,600	40,300	62,200	90,000
Depth of Girder (mm)	1,960	2,640	3,540	4,260
Thickness of Web (mm)	10	10	12	14
Width of Flange (mm)	500	520	540	600
Thickness of Flange (mm)	30	36	36	36
Stress （N/mm^2）	137	137	136	138
Steel weight (ton)	57.7	99.1	158.1	239.7

5. Structural Performance and the Reliability Analysis

5.1 Traffic-load resistant Performance

Traffic-load resistant performance of the girder and its examination code are marshaled as shown in Table-8.

Table-8 Definition of Structural Performance

Traffic load resistant performance	Structural material must be within elastic range and its characteristics are not changed, in the usual state.
Examination code	Possibility of yielding of the material must be small enough. (equal or smaller than the allowable stress design)

5.2 Probability Distribution of yielding stress of Steel material

The allowable stress of SM400 steel is regulated, based on the yielding stress divided by about 1,7. The variance of the yield stress of actually supplied steel material in Japan is researched and reported in the reference 3). The probability distribution characteristics can be set using this data.

Table-9 Probability Characteristics SM400 (N/mm2)

	Measured 1967	Measured 2002
Mean (μ)	266.6	296.3
SD (σ)	17.54	22.77
V （σ / μ）	0.066	0.077

Figure-9 was described as Normal distributions by these reported yield stress data.

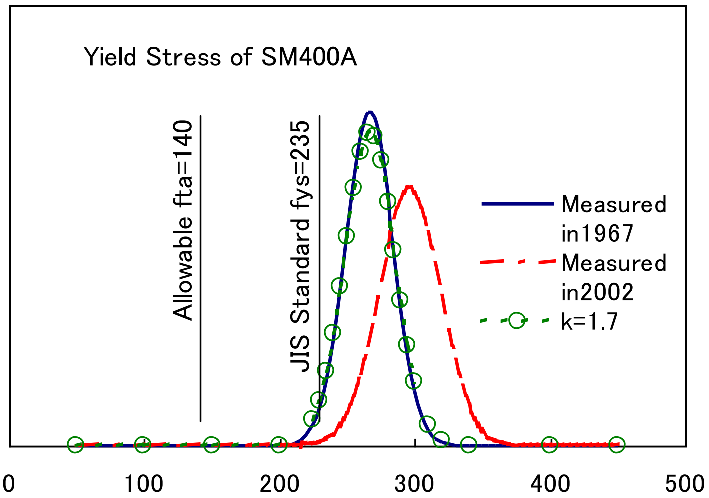

Figure-9 Distribution of Yield Stress (N/mm2)

The relational expression between nominal standard value and the probability characteristics is described as below.

$$f\mu y = \frac{f_{ys}}{1 - k \cdot V_{Ry}} \quad \cdots (4)$$

$f\mu y$: Mean value of yield stress

f_{ys} : JIS standard value of yield point

k : Coefficient for shifting

V_{Ry} : Coefficient of variance

Circle points of Figure-9 are on the fitting line of the coefficient of value is 1.7, and value is 7%. It means that these coefficients are applicable for the reliability analysis based on the yield stress data measured in 1967.

5.3 Reliability Index

In order to examine the Traffic-load resistant performance, Reliability Index based on second moment equation is introduced.

$$\beta = \frac{f\mu y - (f\mu d + f\mu p1 + f\mu p2)}{\sqrt{(f\mu y \cdot V_{Ry})^2 + (f\mu d \cdot Vd)^2 + (f\mu p1 \cdot Vp1)^2 + (f\mu p2 \cdot Vp2)^2}} \quad \cdots \quad (5)$$

β : Reliability Index
$f\mu d$: Mean value of fiber stress by Dead Load
$f\mu p1$: Mean value of fiber stress by Live Load $p1$
$f\mu p2$: Mean value of fiber stress by Live Load $p2$

Vd : Coefficient of variance of Dead Load
$Vp1$: Coefficient of variance of Live Load $p1$
$Vp2$: Coefficient of variance of Live Load $p2$

$$f\mu p1 = \frac{M\mu p1}{Z_g} \qquad (M\mu p1 = \mu p1 \cdot I_{p1}) \quad \cdots \quad (6)$$

$\mu p1$: Mean value of Live Load $p1$
$M\mu p1$: Bending Moment by $\mu p1$
Z_g : Section modulus of the Girder
I_{p1} : Volume of influence plane corresponding to $p1$

$$f\mu p2 = \frac{M\mu p2}{Z_g} \qquad (M\mu p2 = \mu p2 \cdot I_{p2}) \quad \cdots \quad (7)$$

$\mu p2$: Mean value of Live Load $p2$
$M\mu p2$: Bending Moment by $\mu p2$
I_{p2} : Volume of influence plane corresponding to $p2$

The addition rule is herein utilized to above calculations, and reliability index of the girder's stress is calculated by adding the dead load case and live load case.

〔Addition Rule for Discrete value〕

When the discrete variables x and y have their mean value [$\mu x, \mu y$] and standard deviations [σ_x, σ_y], $x + y$ has its mean value $\mu x + \mu y$ and standard deviation $\sqrt{\sigma_x^2 + \sigma_y^2}$.

To know the possibility of yielding, the relational expression between the failure possibility and the reliability index can be applied.
The probability of Failure, the next equation;

$$Pf(\beta) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta} e^{\frac{-t^2}{2}} dt \quad \cdots (8)$$

can be applied. And the numeric values are derived and shown in Table-10.

Table-10

β	3.0	4.0	5.0	6.0
Pf	1.4×10^{-3}	3.2×10^{-5}	2.9×10^{-7}	9.9×10^{-10}

Reliability assessment for the same Bridge as designed in section 4.2 is carried out, using the equation-5. The characteristics of the distribution and the calculated reliability Indexes are shown in Table-11. The variance of the dead load was set to zero in this assessment, and the traffic load characteristics (, etc.) were selected from the Traffic Flow C.

Table-11 Reliability Assessment (N/mm^2)

Span (m)		30	40	50	60
Dead Load	$f\mu d$	78.1	84.9	89.7	95.2
Live Load p1	$f\mu p1$	22.0	17.6	14.3	12.0
	$Vp1$	0.486	0.486	0.486	0.486
Live Load p2	$f\mu p2$	16.9	17.4	17.5	17.5
	$Vp2$	0.509	0.441	0.394	0.360
Yield Stress (SM400A)	$f\mu y$	267			
	V_y	0.07			
Reliability Index β		6.4	6.7	6.9	6.9

In this Table-11, the reliability index gradually increases along the increase of span length. That is due to the reduction of the live-load share.
In the case of 30m span, the probability distributions of each load share and the yield stress are shown in Figure-10.

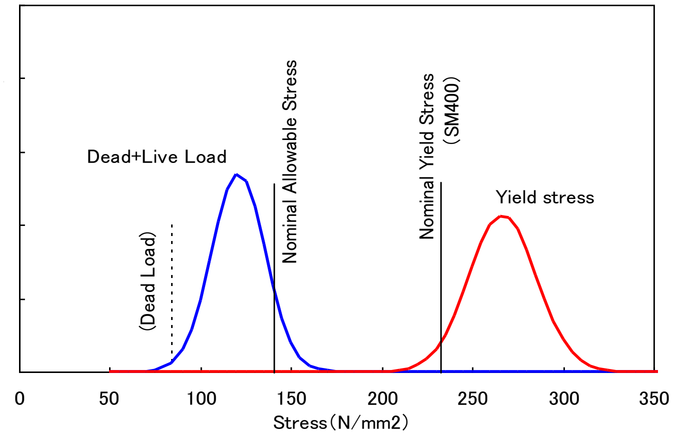

Figure-10 Probability distributions of Reliability Assessment (Traffic Flow C)

On the other hand, with respect to the Reliability Analysis, a design principal is introduced that the Reliability Index must be greater than 6.0.
The results are shown in Table-12.

Table-12 Reliability Assessment of Redesigned Girder (N/mm^2)

Bridge Span (m)		30	40	50	60
Dead Load	$f\mu d$	81.5	91.8	98.9	104.2
Live Load p1	$f\mu p1$	22.9	19.0	15.8	13.2
	$Vp1$	0.486	0.486	0.486	0.486
Live Load p2	$f\mu p2$	17.6	18.9	19.3	19.1
	$Vp2$	0.509	0.441	0.394	0.360
Yield Stress (SM400A)	$f\mu y$	267			
	V_y	0.07			
Reliability Index β		6.1	6.1	6.2	6.2

In this Table-12, the reliability indexes are all nearly equal to 6.0.
Because the dimension of the girder is redesigned as shown in Table-13. The steel weight of the girder is reduced about 3% compared with Table-7.

Table-13 Dimension of Redesigned Girder

Bridge Span (m)	30	40	50	60
Depth of Girder (mm)	1,960	2,630	3,540	4,260
Thickness of Web (mm)	10	10	12	14
Width of Flange (mm)	460	500	500	540
Thick of Flange (mm)	31	34	34	35
Steel Weight (ton)	56.0	93.7	148.5	227.0

6. Aspect of Uncertainty

As described above, allowable stress design method can be converted to the Reliability Analysis. This is a comprehensive conversion using the usual information. This analysis is based on the premise that the bridge girder was designed without mistake and has no defects in the structure material. The probability distribution of the acting stress and that of the yielding strength which is inherent in the base metal are used for the calculation of Reliability Index.

In the actual bridge structure, aged damage or the defects of welding probably exist, and the error of design or the difference from the designed dimension is underlying. At this time, these uncertainty factors are not verified numerically. Then a simplified assumption is introduced, and numerical simulation is carried out through the Random Number Generation. The span length 30m is adopted for the below simulation example.

6.1 Deterioration Factor

The deterioration of the material is assumed as follows;

$$\boldsymbol{fr} = \boldsymbol{fy} \cdot \boldsymbol{Cr} \qquad \cdots \quad (9)$$

$$\begin{cases} \boldsymbol{fr} & \text{: deteriorated strength of the material} \\ \boldsymbol{fy} & \text{: yield strength of the sound material} \\ \boldsymbol{Cr} & \text{: coefficient of deterioration} \end{cases}$$

These variables have their individual probability distribution. The deteriorated strength $\boldsymbol{fr}$ is defined as a reduced value of the sound material strength, and has the probability distribution of the product of sound strength and coefficient of deterioration. The distribution property of the sound material is defined as the mean $\boldsymbol{f\mu y}$ and variance $\boldsymbol{V}_y$. They are same as equation-4.

Coefficient of deterioration $\boldsymbol{Cr}$ is greater than zero and smaller than 1.0, and its frequency gradually decreases with closing to zero. This is based on an assumption that the healthiness in the immediate state must be greater than the aged stage.

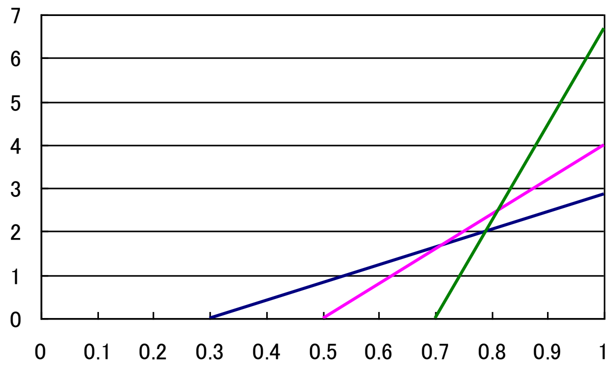

Figure-11 Probability Distribution Image of $\boldsymbol{Cr}$

The probability distribution examples of ***Cr*** are shown in Figure-11. Every area of these triangles equals to 1, and intersection points to horizontal axis mean the minimum value of ***Cr***. Arbitrary gradient can be adopted depending on the deterioration degree.

If ***Cr*** smaller than 0.5 is frequent, many a bridge may collapse. Such an accident is not so frequent. As a realistic presumption, the red triangle was chosen and the random samples of ***Cr*** are generated for simulating the fragility distribution. Then the deteriorated strength ***fr*** is derived by equation-9.

Figure-12 shows the histogram of the calculated ***fr*** and the yield strength distribution of the sound base metal ***fy*** (blue line).

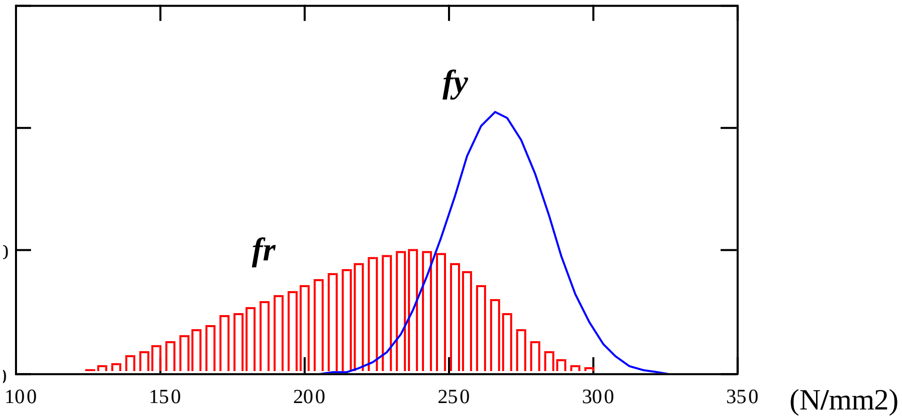

Figure-12 Histogram of ***fr*** and sound Yield Stress Distribution ***fy***

Table-14 Distribution Properties of ***fy*** **and** ***fr*** (N/mm^2)

fy (sound yield strength)	μ (mean)	267
	σ (standard deviation)	17.5
fr (deteriorated strength)	μ (mean)	220.2
	σ (standard deviation)	36.0

Deteriorated strength ***fr*** has the properties of reduced mean value (18%) and diffused standard deviation value (200%).

Reliability assessment using this fragility curve and the acting stress distribution is carried out. Figure-13 shows the histogram of acting stress and deteriorated strength.

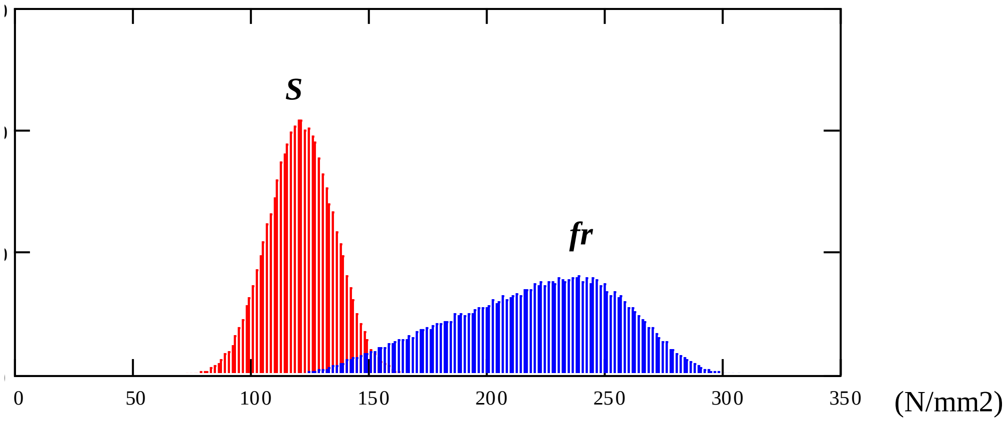

Figure-13 Histogram of acting stress ***S*** and deteriorated strength ***fr***

Using these histogram ordinates, numerical convolution can be derived as follows;

$$Pfx = \sum_{i=1}^{n} \left(\frac{Ys_i}{N} \cdot \frac{Yr_i}{N} \right) \qquad \cdots \quad (10)$$

- Pfx : probability of failure for deteriorated stress
- Ys : ordinates of acting stress
- Yr : ordinates of deteriorated strength
- N : number of generated samples

Reliability index and probability of failure cane also be calculated by the second moment method suing equation-5 and equation-8.
Calculated probability and Reliability Index corresponding to that probability are shown in Table-15.

Table-15 Probability and Reliability Index

	convolution	**2nd moment method**
Probability of Failure Pfx	**7.3×10^{-4}**	**5.6×10^{-3}**
Reliability Index β	**3.2**	**2.5**

The second moment method gives the larger probability than the convolution, because the distribution shape of the deteriorated strength is deformed from the symmetrical normal distribution. The convolution results are reasonable.

6.2 Error Factor

The acting stress of the bridge girder is calculated through the load setting and the cross section calculation. If a mistake exists in those procedures, the result of the design is error. This is the uncertainty factor. For example, if the thickness of the flange is thicker than the correct one the acting stress becomes small. And in the reverse case, it becomes opposite result. When the wrong construction was done, the acting stress

becomes wrong, even though the design was correct. Such an uncertainty does not always increase the acting stress.
The diffusion of the acting stress caused by the error factor is assumed as follows;

$$\boldsymbol{Se} = \boldsymbol{S} \cdot \boldsymbol{Ce} \qquad \cdots \quad (11)$$

$\boldsymbol{Se}$: acting stress diffused by error
$\boldsymbol{S}$: correct acting stress
$\boldsymbol{Ce}$: coefficient of diffusion caused by error (normal distribution μ =1.0 , σ =0.2)

Coefficient $\boldsymbol{Ce}$ is assumed to be the normal distribution which has 1.0 mean and 0.2 standard deviation. Normal distribution is the most commonly used distribution. And the standard deviation is adopted because it is unbelievable that $\boldsymbol{Ce}$ exceeds 2.0. Moreover this is the trial simulation to know the numerical effect where we have no evidence-based data. The diffused acting stress $\boldsymbol{Se}$ is calculated, and its histogram is shown in the Figure-14 with the histogram of the correct acting stress (green line).

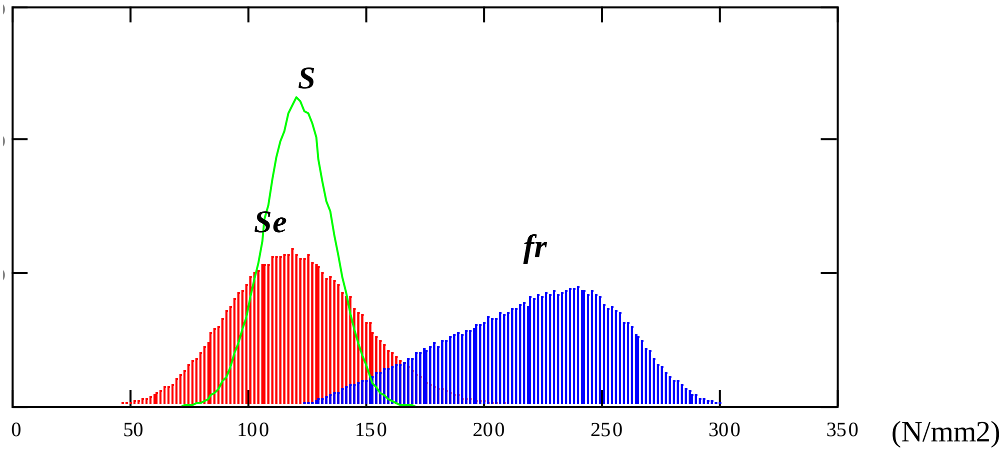

Figure-14 Calculated Histogram of Diffused acting stress $\boldsymbol{Se}$ and deteriorated strength $\boldsymbol{fr}$

The convolution of these histograms $\boldsymbol{Se}$ and $\boldsymbol{fr}$ is carried out in the same as Equation-10, and its results are shown in Table-16. The probability of failure becomes 2.5 times larger than the case without error effect.

Table-16 Probability and Reliability Index for diffused acting stress by error

	convolution
Probability of Failure Pfx	$\mathbf{1.7 \times 10^{-3}}$
Reliability Index β	**2.9**

7. Conclusion

Reliability analysis as an examination method for traffic load resistant performance of a bridge is described above. The characteristics of the variance of the Live Load are focused, from the view point that the dominant load case is "Dead Load + Live Load " for bridge girders. And the acting stress distribution of the girder is derived based on linear addition Rule. The probability distribution of the yield point stress is set based on the reported value. Then the Reliability Index is derived using second moment formula. Two peaks and their variances are comprehensively overviewed, because of convenience and usefulness. That is for engineers who are unfamiliar with the reliability theory but skilled in the allowable stress design.

The reliability assessment without uncertainties for a bridge which is designed in accordance with the allowable stress method showed the reliability index about 6.0. It can be recognized that the allowable stress method provides a sufficiently safe solution. Then the uncertainties such as the deterioration of the steel material and the error factor are assumed and the reliability assessment for those is carried out.

In the case where the uncertainties are considered, the results draw the following points of assessment;

1) When only the deterioration factor is introduced, the Reliability Index becomes 3.2 and it is barely safe.
2) When the error factor is added to above, the Reliability Index becomes 2.9 and it is slightly dangerous.
3) The background of the allowable stress design method implicitly contains some uncertainties.

The actual degree of deterioration depends on the quality control of the construction and the environment condition, or its history. The error factor is influenced by the checking framework in the construction stage. There may be some differences in national culture on these factors. The deterioration factor and the error factor described in this paper are the trial simulation and have no evidence-based data. But we can get a numerical and practical reason for helping the engineering judgment. The measured data is indispensable for ensuring the foundation of the Reliability. But the Reliability Assessment without the uncertainty factor can be accepted, until the evidence-based data is clarified.

The role of the infrastructure responds to the social demands. From that standpoint, traffic flow is the most important factor for Bridge designing. Social role of a bridge is to meet the traffic demand. The width and the geometric alignment of the bridge are decided based on the traffic grade. Therefore, traffic load should be decided considering the traffic grade in the same sense. When the heavier traffic grade is expected than the standard one, the bridge must be strengthened in accordance with that. And, if the percentage of the heavy vehicle is expected smaller than the standard, the bridge must economically be designed.
The allowable stress design method is widely rooted as a de-facto standard. That is the comparison gauge for the new reliability analysis. Many a engineer shall be able to address the reliability design by an accessible method. Then the reliability technology shall develop higher being used by many engineers, and make a social contribution greatly.

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