Structural design with polymorphic uncertainty models

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Abstract. The design of structures is one of the major tasks for engineers. Structural design should be robust with respect to the polymorphic nature and characteristic of the available information. Generally, the availability of information in engineering practice is limited. Incomplete, fragmentary, diffuse, and frequently expert specified knowledge leads to imprecision in data. In addition, engineers have to cope with the objective variability and fluctuations in material, geometry and loading. Uncertainties inherently present in resistance of structural materials, environmental and loads, boundary conditions, physical and numerical models, and to other types of aleatoric and epistemic uncertainties. The goal of numerical structural design, computing robust and reliable structures, can be realized by means of analyzing different variants, application of optimization tasks, or solving inverse problems. The contribution presents selected research results recently obtained by the authors. The main focus is on soft computing methods developed for structural design.

Keywords: structural analysis, structural design, reliability assessment, polymorphic uncertainty

1. Introduction

The main point of view for this contribution are soft computing methods for structural design considering uncertainty. The realistic consideration of the uncertainty of the underlying database is essential in order to compute realistic results. Uncertainty can be classified into aleatoric and epistemic uncertainty, see e.g. (Möller and Beer, 2008) and (Muhanna et al., 2007). The uncertainty characteristic variability is of aleatoric manner. Epistemic uncertainty can be separated into incompleteness and impreciseness. An uncertain variable, which should be considered in structural analysis, is mostly characterized by more than one of these characteristic. Therefore, the aim of current research is to describe all uncertainty characteristics with one model, see e.g. (Reuter et al., 2012). These polymorphic uncertainty models provide the possibility for a realistic investigation of structural behavior and yields to better results in design processes. The numerical analysis based on two independent algorithms, fuzzy analysis and stochastic analysis in sequential applications.

Two different approaches for considering uncertainty in optimization tasks were developed. On the one hand, there is the *wait-and-see* strategy, see (Dantzig, 1955), also known as passive approach. On the other hand, there the *here-and-now* strategy, see (Tintner, 1960) and (Sengupta et al., 1963), denoted as active approach. These approaches mainly differ in the order of applying structural optimization and estimation of uncertainty. The known investigations and applications for the two approaches, see (Wets, 2002), are using especially random variables to describe the uncertainty of data. In this contribution approaches for considering polymorphic uncertainty in optimization tasks are shown. The importance of considering polymorphic uncertainty for structural analysis and design is demonstrated by means of an example.

2. Polymorphic uncertainty modelling

Two well established uncertainty models – random variables and fuzzy sets – are the basis for polymorphic uncertainty variables, see (Beer et al., 2012). In the following, the two basis models are explained and the polymorphic uncertainty models are defined as well.

2.1. Basic uncertainty models

2.1.1. *Random variables*

A random variable is a mapping $X\colon \Omega \to \mathbb{R}$ that satisfies the condition

$$\forall\, I \in \mathcal{B}(\mathbb{R})\colon X^{-1}(I) := \{\omega \in \Omega;\; X(\omega) \in I\} \in \Sigma. \tag{1}$$

Thereby is (Ω, Σ, P) a probability space and $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ an observation space. Ω is the set of elementary events ω and Σ is a σ-Algebra. The probability measure P has to satisfy the axioms of Kolmogorov. The notation $\mathcal{B}(\mathbb{R})$ is the Borel-σ-Algebra. For the random variable X, the associated probability measure P_X is defined as

$$P_X\colon \mathcal{B}(\mathbb{R}) \to [0,1]\colon I \mapsto P_X(I) = P\left(X^{-1}(I)\right). \tag{2}$$

2.1.2. *Fuzzy sets*

A fuzzy number extends the entropy of a crisp subset $A \subseteq \mathbb{R}$. This extension is done by an evaluation of the vector space $\mathbb{R}$ with membership functions $\mu\colon \mathbb{R} \to [0,1]$ and can be written as set of ordered pairs

$$\{(x, \mu(x)) \mid x \in \mathbb{R} \wedge \mu(x) \in [0,1]\}. \tag{3}$$

The set of all fuzzy sets of $\mathbb{R}$ is denoted by $\mathcal{F}(\mathbb{R})$. For numerical implementation it is necessary to introduce α-level cuts for fuzzy variables. For a convex fuzzy variable $\tilde{A}$ and $\alpha \in (0,1]$,

$$\tilde{A} := \{x \in \mathbb{R}\colon \mu_{\tilde{A}}(x) \geq \alpha\} \tag{4}$$

is called α-level cut.

2.2. Polymorphic uncertainty models

The main idea of polymorphic uncertainty is the consideration of more than one uncertainty characteristic by one variable. Polymorphic uncertainty models are introduced in (Möller and Beer, 2004) and extended in (Pannier et al., 2013); an overview is given in Table I and (Beer et al., 2012). The definition of a polymorphic uncertain variable is based on the definition of random variables, whereby parts of this definition are expressed by fuzzy numbers.

2.2.1. *Fuzzy probability based randomness (fp-r)*

A type of polymorphic uncertainty modelling is a fuzzy random based random variable, which can take variability and incompleteness into account. For fuzzy random based random variables, the probability measure P of the random number, see Eq. (2), is defined as an evaluated set of probability functions. This means, every event is represented by a fuzzy value and not by a single number. The fuzzy probability space is the triple $(\Omega, \Sigma, \hat{P})$. Ω and Σ are equal to the random number definition. The fuzzy probability $\hat{P}$ is a family of mappings

$$\hat{P} = \left(\hat{P}_\beta\right)_{\beta \in (0,1]}, \tag{5}$$

where $\hat{P}_\beta$ assigns to each $A \in \Sigma$ an interval $[\hat{P}_{\beta,\mathrm{l}}(A), \hat{P}_{\beta,\mathrm{r}}(A)]$, such that

$$0 \leq \hat{P}_{\beta,\mathrm{l}}(A) \leq \hat{P}_{\beta,\mathrm{r}}(A) \leq 1 \tag{6}$$

holds, here β indicates an α-level. The relating measurable mapping $X\colon \Omega \to \mathbb{R}$ is called fuzzy random based random variable. To describe a cumulative distribution function (cdf), it is necessary to define the set

$$\begin{aligned} \forall (F_X)_\beta \;:=\; & \{G\colon \mathbb{R} \to [0,1] \text{ cdf} \mid \forall x \in \mathbb{R}\colon \\ & \hat{P}_{\beta,l}\left(X^{-1}((-\infty, x])\right) \leq G(x) \leq \hat{P}_{\beta,r}\left(X^{-1}((-\infty, x])\right)\}. \end{aligned} \tag{7}$$

The fuzzy cumulative distribution function is the family of the sets

$$F_X = ((F_X)_\beta)_{\beta \in (0,1]} \,. \tag{8}$$

For the family F_X, a fuzzy set $\tilde{s} = (\tilde{s}_\beta)_{\beta(0,1]}$ can be defined, such that for each $\beta \in (0,1]$ and $s \in \tilde{s}_\beta$ F_s is a unique cumulative distribution function. The fp-r variable F_X can be written by a bunch parameter representation

$$F_X = (\{F_s \mid s \in \tilde{s}_\beta\})_{\beta \in (0,1]}\,, \tag{9}$$

see also (Möller and Beer, 2004). A visualization of the fuzzy cumulative distribution function of a fp-r variable is shown in Figure 1.

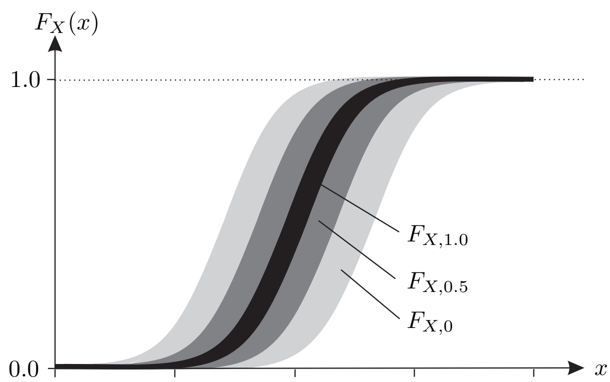


Figure 1.: Fuzzy random based random variable (fp-r)

2.2.2. *Fuzzy probability based fuzzy randomness (fp-fr)*

Fuzzy random based fuzzy random variables consider the uncertainty characteristics variability, incompleteness and imprecision. Taking imprecision additionally to fp-r into account, it is necessary to define a σ-algebra $\mathcal{B}(\mathcal{F}(\mathbb{R}))$, yielding to the measurable space $(\mathcal{F}(\mathbb{R}), \mathcal{B}(\mathcal{F}(\mathbb{R})))$ for fuzzy numbers. With the already introduced fuzzy probability space $(\Omega, \Sigma, \hat{P})$, it yields the measurable mapping

$$X\colon \Omega \rightarrow \mathcal{F}(\mathbb{R}), \tag{10}$$

called fuzzy random based fuzzy random variable. The fp-fr variable X can be formulated as family of fuzzy probability intervals X_α. If for every $\omega \in \Omega$, the fuzzy value $X(\omega)$ is convex, then each α-level is an interval. It holds

$$X(\omega) = (X(\omega)_\alpha)_{\alpha\in(0,1]} = (X_\alpha(\omega))_{\alpha\in(0,1]} = ([X_{\alpha,l}(\omega), X_{\alpha,r}(\omega)])_{\alpha\in(0,1]}\,, \tag{11}$$

with $X_{\alpha,l}(\omega)$, $X_{\alpha,r}(\omega) \in \mathbb{R}$ and $X_{\alpha,l}(\omega) \leq X_{\alpha,r}(\omega)$. The distribution of the fp-fr variable X is written as family of distribution functions

$$F_X = (F_{X,\alpha})_{\alpha\in(0,1]} := ([F_{X,\alpha,l}, F_{X,\alpha,r}])_{\alpha\in(0,1]}. \tag{12}$$

The interval $[F_{X,\alpha,l}, F_{X,\alpha,r}]$ characterizes the distribution of the fp-r interval X_α. The distribution of $X_{\alpha,l}$ and $X_{\alpha,r}$ is defined as

$$F_{X,\alpha,l} := ((F_{X,\alpha,l})_\beta)_{\beta\in(0,1]}\,, \text{ and } F_{X,\alpha,r} := ((F_{X,\alpha,r})_\beta)_{\beta\in(0,1]}\,. \tag{13}$$

A visual representation of a fp-fr variable X can be seen in Figure 2.

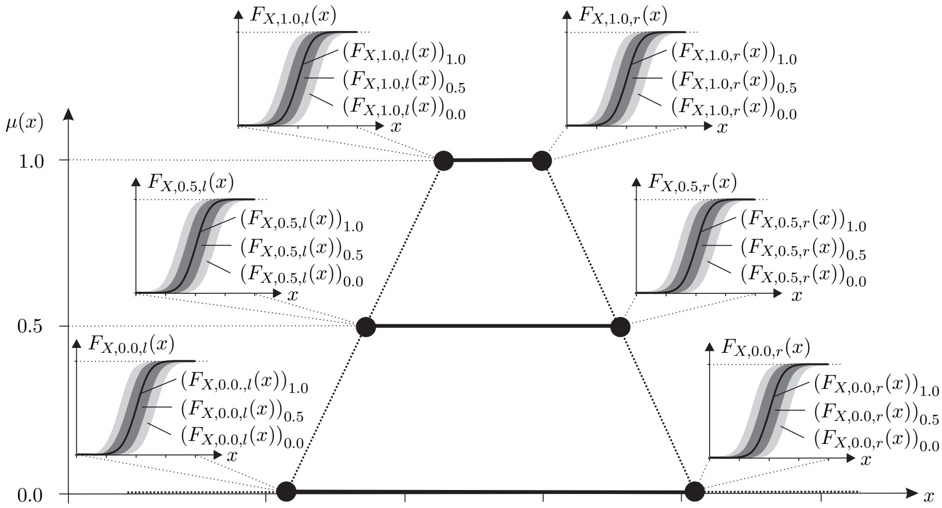


Figure 2.: Representation of a fuzzy random based fuzzy random variable (fp-fr)

Table I. Overview of polymorphic uncertainty models (Pannier et al., 2013)

name	mapping	data	characteristic
random variable	$X\colon \Omega \to \mathbb{R}$ $P\colon \Sigma \to [0,1]$	deterministic very many	aleatoric (variability)
fuzzy variable	$\mu_{\tilde{A}}\colon \mathbb{R} \to [0,1]$	subjective, few	epistemic
fuzzy random variable	$X\colon \Omega \to \mathcal{F}(\mathbb{R})$ $P\colon \Sigma \to [0,1]$	imprecise (very) many	variability, impreciseness
fuzzy random based random variable (fp-r)	$X\colon \Omega \to \mathbb{R}$ $\hat{P} = \left(\hat{P}_\beta\right)_{\beta\in(0,1]}$	deterministic some	variability, incompleteness
fuzzy random based fuzzy random variable (fp-fr)	$X\colon \Omega \to \mathcal{F}(\mathbb{R})$ $\hat{P} = \left(\hat{P}_\beta\right)_{\beta\in(0,1]}$	imprecise some	variability, impreciseness, incompleteness

2.3. Numerical analysis concepts with uncertain data

A generalized formulation of a set of polymorphic uncertain variables $\mathcal{D}(Q,R)$ allows a mathematical formulation for all mentioned uncertainty measures. $\mathcal{D}(Q,R)$ means, e.g. $\mathcal{D}(\Omega,\mathbb{R})$ is a set of random numbers, respectively, $\mathcal{D}(\mathbb{R},[0,1])$ is a set of fuzzy numbers. The notation is an abbreviation, which always implements further relating properties. The application of polymorphic uncertainty models needs specific numerical algorithms. The basic uncertainty models and the other polymorphic uncertainty models are included as special case. In general, the algorithm is separable into fuzzy analysis and stochastic analysis. The fuzzy analysis handles the fuzzy properties of the

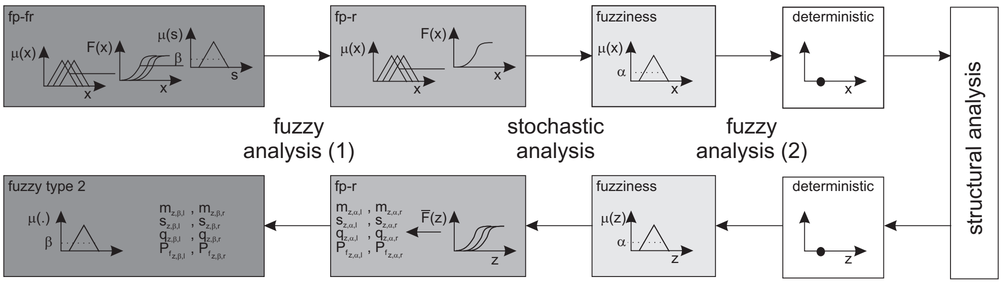


Figure 3.: Sequential reduction of uncertainty

considered uncertainty model, applying the well known α-level optimization, see e.g. (Möller and

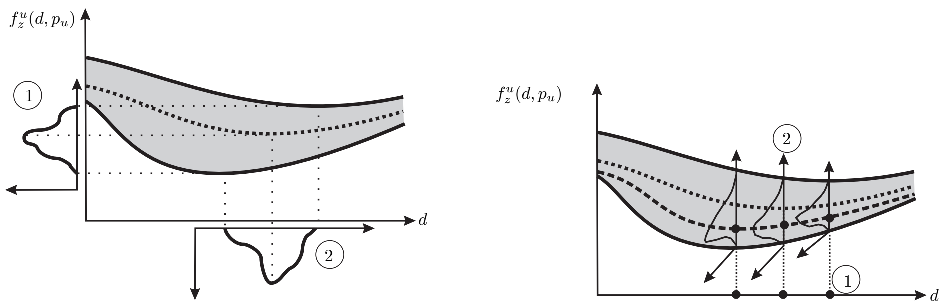
Beer, 2004). The stochastic analysis takes the uncertainty characteristic variability into account. It is evaluated by standard or advanced Monte Carlo simulation (MCS). The sequential reduction of uncertainty is shown in Figure 3. The application of the first fuzzy analysis evaluates the incompleteness characteristic by α-level optimization at β level. The variability of the resulting fuzzy random based random variable is evaluated by MCS, with the aim to compute e.g. probability of failure P_f, empirical mean values m, or empirical variances s. The resulting fuzzy set is again handled by α-level optimization at α level to evaluate the imprecision characteristic. The output quantities z of the deterministic structural analysis (e.g. Finite Element Analysis (FEA)) are computes for the input values x. The second fuzzy analysis computes the membership function $\mu(z)$ of all output quantities. The stochastic analysis results in empirical fuzzy distribution functions. These fuzzy distribution functions needs to be evaluated and reduced to an deterministic value, in order to evaluate the fuzzy quantity at β level with the help of the first fuzzy analysis. The evaluation could result in e.g. a fuzzy probability of failure or a fuzzy mean value. These fuzzy quantities are the basis for the first α-level optimization, which evaluates the fuzzy quantities at each α-level to get the information for the resulting β-level. The result can be interpreted as fuzzy type-2 value, see (Wu and Mendel, 2007).

3. Numerical design concepts with uncertain data

In this section are shown numerical design concepts, applying surrogate models, for the consideration of data uncertainty , according to a main contribution, see (Pannier, 2011). Two concepts for numerical design are existing

- optimization
- solution of the inverse problem

This contribution describes surrogate models for the optimization task. For the solution of the inverse problems, see e.g. (Götz et al., 2013).



(a) Passive optimization approach (b) Active optimization approach

Figure 4.: Surrogate models for solving the "uncertain" optimization

The optimization objective function under consideration of uncertain quantities is written as

$$\hat{f}_Z^u : \mathcal{D}(Q,R) \rightarrow \mathcal{D}(V,W). \tag{14}$$

The uncertain input quantities $\mathcal{D}(Q,R)$ can be classified into uncertain design variables $x_d^u \in \mathcal{D}(Q_x,R_x)$ and uncertain a priori parameters $p^u \in \mathcal{D}(Q_p,R_p)$, holding

$$\mathcal{D}(Q_x,R_x) \times \mathcal{D}(Q_p,R_p) = \mathcal{D}(Q,R). \tag{15}$$

An optimization algorithm for the direct consideration of polymorphic uncertain design variables does not exist. This means, that the application of uncertain design variables is directly not possible. A solution is the application of an affine transformation $\mathcal{T}$ to the uncertain design variables, split them to deterministic design variables $d \in \mathbb{R}^{n_x}$ (suitable for optimization algorithms) and further (constant) uncertain a priori variables, containing to the set p^u. The affine transformation has to be defined for each uncertainty model; e.g. for fuzzy quantities

$$\mathcal{T}^f : \mathbb{R}^{n_x} \to \mathcal{F}(\mathbb{R}^{n_x}) : d \mapsto (c(d) \cdot u + d) = \tilde{x}_d \tag{16}$$

can be found. The objective function Eq. (14) can be rewritten as

$$f_Z^u : \mathbb{R}^{n_x} \times \mathcal{D}(Q_p,R_p) \to \mathcal{D}(V,W) : (d,p^u) \mapsto \hat{f}_Z^u(\mathcal{T}(d),p^u). \tag{17}$$

Due to the objective function Eq. (17), considering deterministic design parameters, the "uncertain" optimization task is formulated as

"determine $L \subseteq X_d^+$ such that $\forall\, d \in x_d^+$ and $d_{min} \in L$ it holds:
$f_Z^u(d_{min},p^u)$ is **"lower or equal than"** $f_Z^u(d,p^u)$".

The objective is to find

$$\underset{d \in X_d^+}{\textbf{minimum}}\; f_{\mathcal{Z}}^u(d,p^u), \tag{18}$$

under consideration of the uncertain permissible range

$$\begin{aligned} X_d^+ = \{d \in \mathbb{R}^{n_x} \mid \forall\, j \in \{1;\ldots;a_g\} : \\ g_j^u(d,p^u) \text{ \textbf{"lower than"} } 0 \wedge \forall\, k \in \{1;\ldots;a_h\} : h_k^u(d,p^u) \text{ \textbf{"equal"} } 0\}, \end{aligned} \tag{19}$$

with the uncertain constraints

$$g_j^u : \mathcal{D}(Q,R) \to \mathcal{D}(V,W) \text{ and } h_k^u : \mathcal{D}(Q,R) \to \mathcal{D}(V,W). \tag{20}$$

Due to the missing of a general rule for comparing uncertain quantities, relational operators as $<$, $>$, $=$, used in the introduced design task and the related constraints. The application of the minimum-operator "min" as for the deterministic case in Eq. (18) is not possible. This means, the solution of the design task with uncertain quantities, can be found for surrogate problems only. The surrogate problem can be formulated as passive or active approach. The both approaches mainly differ in the order of evaluation of uncertainty task and solving the optimization problem.

3.1. Passive optimization approach

The passive approach is extended for polymorphic uncertain variables and formulated as

”determine for all p (realization of p^u) that set $L \subseteq X_d^+$, such that
$\forall d \in X_d^+$, and $d_{min} \in L$ it holds: $f_{\mathcal{Z}}(d_{min}, p) \leq f_{\mathcal{Z}}(d, p)$”.

With this approach, the realizations of uncertain a priori parameters p^u will be computed firstly, yielding deterministic objective function, e.g. the dotted line in Figure 4(a). The mapping of uncertainty input quantities to uncertain output quantities is with known approaches only possible, if the affine transformation, Eq. (16), is linear dependent from the design variable. The next step is the computation of the minimum of these deterministic objective functions with deterministic optimization techniques, and has to be repeated for all necessary realizations of p^u. The direct result of this computation is a set of the minima of all deterministic functions and could be interpreted as the minimum of the uncertain objective function, see Figure 4(a), 1. In a pre-processing step, the single points of this uncertain minimum can be associated with the related design quantity, yielding to an uncertain evaluation for the design quantities, (Figure 4(a), 2). The checking of the permissibility for the uncertain set of designs has to be done afterwards. A scheme of the complete algorithm can be seen in Figure 5.

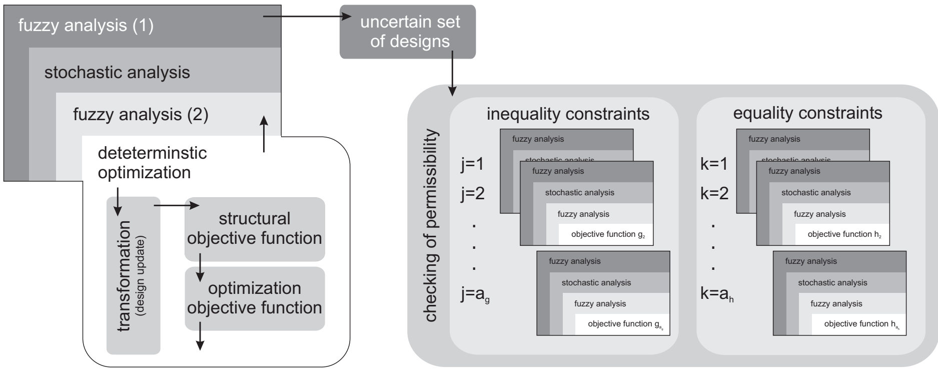


Figure 5.: Framework for computing the optimization task with the passive approach

3.2. Active optimization approach

The formulation of the active approach for polymorphic uncertain quantities is

”determine $L \subseteq X_d^+$ such that
$\forall\, d \in X_d^+$ and $d_{min} \in L$ it holds: $\mathcal{M}\left(f_{\mathcal{Z}}^u(d_{min}, p^u)\right) \leq \mathcal{M}\left(f_{\mathcal{Z}}^u(d, p^u)\right)$”.

Selected deterministic design quantities d (Figure 4(b), 1) were transformed to uncertain design quantities x_d^u and the uncertain result quantities (Figure 4(b), 2) can be computed. The application of information reducing measures

$$\mathcal{M} : \mathcal{D}(V, W) \to \mathbb{R} \tag{21}$$

allows the reduction to deterministic values and an application of further optimization processing. There are two different types of information reducing measures. On the one hand, there are measures reducing the uncertainty to an representative value. On the other hand, there are measures quantifying the uncertainty. In dependency of the specific uncertainty model and different design objectives, different information reducing measures could be formulated. Due to the main difference to the passive approach – evaluating the uncertainty in the second step – there are no requirements for an affine transformation. This approach is the basis for established Reliability-Based Optimization methods (RBO), see e.g. (Enevoldsen and Sørensen, 1994).

Summarizing, by means of the passive approach the location of the minimum is detectable, there are no uncertain response parameters, the checking of constraints has to be done in a post processing, but there is a fast detection of optimal ranges. By contrast, with the aid of the active approach it is possible to calculate a deterministic minimum for deterministic design parameters. It includes an early reduction of information. The constraints could be evaluated directly, but this is an expensive detection of the optimum. The combination of both approaches could use the particular advantages. This could be done by a sequential application of the approaches. In an early design stage, the passive approach with the uncertainty model fuzziness should be applied. In the final design stage, the active approach with random based models should be used.

4. Numerical efficiency

The applicability of polymorphic uncertainty models for structural analysis and design is, due to the enormous numerical effort, limited. The consideration of polymorphic uncertainty models makes it necessary to reduce the computational effort. There are three essential possibilities to reduce the numerical effort

- reduction of calculation time for the deterministic solution or reduction of necessary number of computations of the deterministic solution,
- replace the deterministic solution with efficient metamodels,
- parallel evaluation of deterministic solutions.

The reduction of the necessary number of computations of the deterministic solutions, is possible for polymorphic uncertainty models, due to the sequential evaluation of fuzzy properties and stochastic properties, see Section 2, for these two sequences. The stochastic solution can be applied by well known MCS or by more efficient technologies as subset sampling, see e.g. (Au, 2001). The solution for fuzzy quantities can be improved by applying efficient optimization technologies for the α-level optimization, see (Möller et al., 2000); or a specific evaluation of needed points of the membership function.

Common for both is, the numerical effort increases with increasing number of uncertain parameters. This means, reducing the number of uncertain quantities improves the numerical efficiency. This necessary categorization of uncertain quantities into relevant and non-relevant is possible by analyzing the sensitivity for each input quantity to the output quantity of interest, see e.g. (Saltelli et al., 2008). Metamodels are versatile, e.g. there are applicable for pattern classification, function approximation or computing sensitivity measures. The possibility to approximate functional data, a mapping of input quantities to output quantities

$$f^{*}\colon \mathbb{R}^n \supset H^n \to \mathbb{R}^m \tag{22}$$

can be done by several types of metamodels, see (Søndergaard, 2003). The definition of the region of interest $H^n = [a_1, b_1] \times \ldots \times [a_n, b_n] \mid a_i, b_i \in \mathbb{R},\ a_i \leq b_i,\ i \in \{1, \ldots, n\}$ is necessary to avoid extrapolating. The metamodel f^* can be found on the basis of a set of support points $\mathcal{N} = \{(\underline{x}, \underline{z}) \mid \underline{x} \in H^n, \underline{z} \in H^m\}$, which provide the relations between input and output quantities of the original function $f\colon \mathbb{R}^n \to \mathbb{R}^m,\ \underline{x} \mapsto \underline{z}$ point wise. The well established metamodels to approximate functional data, as Artificial Neural Networks (ANN) or Radial Basis Functions Networks (RBFN) need intensive, time-consuming training. Furthermore, an assumption about the network architecture (e.g. number of hidden layers and number of neurons or radial basis functions inside each hidden layer) is necessary. But the quality of the network mainly dependents on this architecture, especially for a small number of neurons. The computation of the best fitting architecture can be done by investigation of different variants, accompanied by high computational costs. To avoid making a wrong assumption about network type and architecture, (Huang et al., 2006) developed a specific training algorithm for ANN and RBFN, called Extreme Learning Machine (ELM). This metamodel needs a short time for training.

5. Example

This example demonstrates the applicability of the active optimization approach, by optimizing a supporting structure for traffic road signs under car crash loading. The optimization is done under consideration of passive safety requirements of the occupants. In Figure 6 the complete simulation model with approx. 25000 elements is shown.

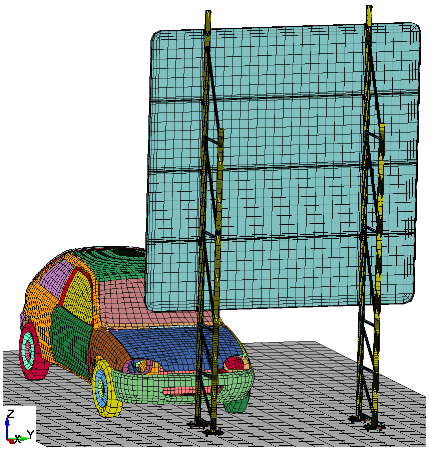


Figure 6.: Simulation model

The simulation model, the uncertain structural analysis and the optimization are discussed in the following.

Simulation model. The car crash against the truss, which is the supporting structure of the traffic sign, is simulated by a Finite Element Model applying explicit time-step integration. The car is modelled by a dummy airbag to capture the acceleration of the head, which is relevant for passive safety requirements. The deterministic simulation results for an impact velocity of $v_{init} = 50\,\mathrm{km/h}$ are shown in Figure 7. In Figure 8 the related filtered acceleration of the head is shown. By integrating the acceleration of the head over time the safety relevant parameter $HIC36$ can be computed. For the shown example it yields $HIC36 = 3.79$, which is under the critical constrain.

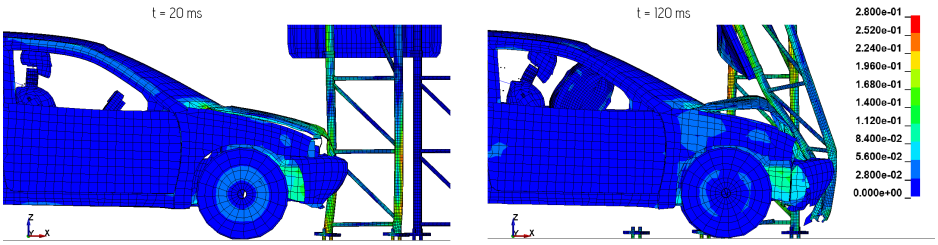


Figure 7.: von-Mises stress ($v_{init} = 50\,\mathrm{km/h}$) at $t = 120\,\mathrm{ms}$

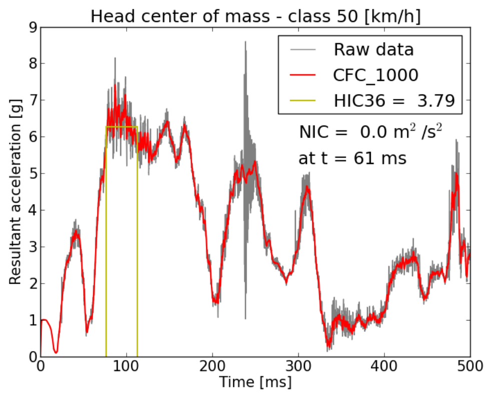


Figure 8.: Acceleration time dependency of the head

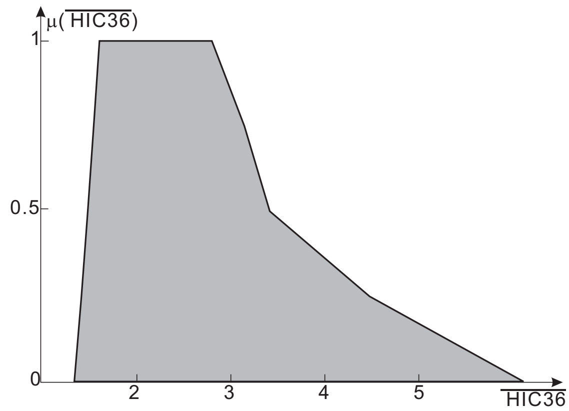


Figure 9.: Fuzzy mean value $HIC36$ of the optimized structure

Uncertainty model. The simulation model is characterized by five uncertain parameters. The car velocity is modelled as fuzzy interval $\tilde{v}_{init} = \langle 35, 50, 90, 100 \rangle\,[\mathrm{km/h}]$. The impact angle $\tilde{\phi} = \langle 45, 45, 90 \rangle\,[°]$ and the uncertain a priori part of the stiffness of the spring at the supporting point (design parameter) $\tilde{K}_u = \langle -20, 0, 20 \rangle\,\mathrm{kN/mm}$ are considered as fuzzy triangular numbers. The welding at the supporting point are modeled as fuzzy probability based random variables. The parameters are normal distributed, with fuzzy mean values.

$$\sigma^{FS} = \mathrm{N} \sim (\mu = \langle 0.355, 0.36, 0.365 \rangle, \sigma = 0.010)\ [\mathrm{kN/mm^2}] \quad (23)$$

$$\epsilon^{FS} = \mathrm{N} \sim (\mu = \langle 0.145, 0.15, 0.155 \rangle, \sigma = 0.005)\ [-] \quad (24)$$

Due to the high numerical effort the acceleration the head ($HIC36$) is approximated by an Artificial Neural Network, based on 100 computed sample points. The result of the fuzzy stochastic analysis is an empirical distribution function, represented by the fuzzy mean of $HIC36$.

Optimization The optimized structure should minimize the injury risk $\min HIC36$ for the dummy. The design parameter is the stiffness of the spring at the supporting point, considering the related a priori uncertainty additive. The design space is the interval $K_d \in [350, 550]$ kN/mm. The result of the uncertain analysis can not be considered directly, due to the non-possible comparison of uncertain parameter. The fuzzy mean of $HIC36$ is mapped to a deterministic value, by applying information reducing measures, to obtain the comparability. The applied information reducing measure is a representative measure (after Rommelfanger). The optimal stiffness of the spring is $K_d = 352.31$ kN/mm. In Figure 9 the related uncertain result of the uncertain analysis is shown.

6. Conclusion

This contribution focuses on soft computing method, developed for structural design under consideration of unavoidable uncertainty. The shown polymorphic uncertainty models allow a realistic consideration of uncertain data, with a separated computation of the different uncertainty characteristics, variability, imprecision and incompleteness. The introduced polymorphic uncertain model fuzzy probability based fuzzy randomness includes the other models as special cases. The numerical implementation is shown.

Furthermore surrogate models, in active and passive manner, for considering polymorphic uncertainty models in design tasks are demonstrated. The split to deterministic design variable and uncertain a priori parameter of the uncertain design variables allows the application of existing optimization algorithms.

Due to the enormous numerical effort, the application of metamodels is seen as unavoidable by the authors.

The applicability of the fuzzy probability based randomness in an active optimization task is demonstrated by means of an example.

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