Decision Making under Interval Uncertainty: What Can and What Cannot Be Computed in Real Time

O. Kosheleva and **V. Kreinovich**

University of Texas at El Paso, El Paso, TX 79968, USA
olgak@utep.edu, vladik@utep.edu

Keywords: *decision making, interval uncertainty, real-time computations.*

Abstract

In engineering, we constantly need to make decisions: which design to select, which parameters to select for this design, etc.

The traditional approach to decision making is based on the assumption that we know all possible consequences of each alternative, and we know the probability of each such consequence. Under this assumption, we can describe a rational decision-making process: to each possible consequence, we assign a numerical values called its utility, and we select the alternative for which the expected value of the utility is the largest.

An important advantage of this approach is that it can be performed in *real time:* if after we made a decision, a new alternative appears, we do not need to repeat the whole analysis again: all we need to do is compare the new alternative with the previously selected one.

In the past, when we used the same procedures year after year, we accumulated a lot of data about the consequences of different decisions – based from which we could estimate the desired probabilities. Nowadays, with new technologies, new materials constantly emerging, we do not have such detailed information about the consequences of these new technologies. As a result, we often only have partial information about the corresponding probabilities. Different possible probability values result in different values of expected utility. Hence, for each alternative, instead of a single value u of expected utility, we have a range (interval) $[\underline{u}, \overline{u}]$ of possible values. We need to make a decision under such interval uncertainty.

The problem is that if two intervals corresponding to two alternatives intersect, then, depending on the probabilities, both can be optimal. One possibility is to return the list of all the alternatives which *can* be optimal. A straightforward computation of this list take time which is quadratic in number of alternatives – and is not possible in real time. We show, however, that it *is* possible to come up with a real-time algorithm for computing the desired list.

Returning the list makes sense if an expert makes the final decision. This makes sense for major decisions, but for numerous minor decisions, it is preferable to automatically select a unique alternative. A natural way to do it is to use Hurwicz optimism-pessimism criterion in which we select an alternative for which, for some α, the value $\alpha \cdot \overline{u} + (1-\alpha) \cdot \underline{u}$ is the largest. When α is fixed, we can find the best alternative in real time. However, when the goal is to produce best alternatives corresponding to all possible values α, then we prove that a real-time algorithm is no longer possible.